**SimBiology®** User's Guide

R2013a

# MATLAB®



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SimBiology<sup>®</sup> User's Guide

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# Contents

# Modeling

What is a Model?	1-2
Model Definition	1-2
Expressions	1-2
Quantities	1-3
Model Hierarchy	1-4
More About	1-4
More About	1-9
Model Modifiers	1-6
Variants	1-6
Doses	1-6
Representing a Model and Model Modifiers	1-7
Constructing a Simple Model	1-7
	1-7
SimBiology Objects	1-7
Model Object	1-9
Objects Representing Quantities	1-10
Scoping of Compartments, Species, and Parameters	1-10
Naming of Compartments and Species	1-10
More About	1-10
Compartment Object	1-11
	1-11
Species Object	1-12
How Species Amounts Change During Simulations	1-12
Keeping a Species Amount Unchanged	1-12
Changing a Species Amount with a Reaction or Rule	1-13
Changing a Species Amount with a Rule When Species is	-
Part of a Reaction	1-14
Keeping a Species Amount Unchanged When Species is	
Part of a Reaction that Adds or Removes Mass	1-14

1

Parameter Object	1-16
Scope of Parameter Objects	1-16
Objects Representing Expressions	1-17
When Reactions, Rules, and Events Specify Parameters	1-17
More About	1-17
Reaction Object	1-18
Writing Reaction Expressions	1-10
Writing Reaction Rate Expressions Explicitly	1-19
Creating Reaction Rate Expressions Using Kinetic Law	1-15
Objects	1-20
Examples of Creating Reaction Rates	1-21
How Reaction Rates Are Evaluated	1-22
Viewing Equations for Reactions	1-24
More About	1-24
Rule Object	1-25
Writing Rule Expressions	1-25
Algebraic Rules	1-26
Initial Assignment Rules	1-27
Repeated Assignment Rules	1-27
Rate Rules	1-28
Examples of Creating Rate Rules	1-29
More About	1-31
Event Object	1-32
Event Triggers	1-32
Event Functions	1 - 33
Specifying Event Triggers	1-33
Specifying Event Functions	1-35
Simulation Solvers for Models Containing Events	1-36
How Events Are Evaluated	1-36
Evaluation of Simultaneous Events	1-38
Evaluation of Multiple Event Functions	1-39
When One Event Triggers Another Event	1-39
Cyclical Events	1-40
Using Events to Address Discontinuities in Rule and	1 10
Reaction Rate Expressions	1-40
Objects Representing Model Modifiers	1-42
More About	1-42

Variant Object	1-43
Creating and Simulating with Variants	1-43
Simulating with Multiple Variants in a Model	1-44
Dose Object	1-45
Creating and Simulating Dose Objects	1-45
Simulation Solvers for Models Containing Doses	1-46
Simulation Solvers for Models Containing Doses	1-40
Create and Simulate a Model That Changes a Species	1 47
Amount	1-47
Overview	1-47
Opening the Example Model	1-48
Adding an Event to the Example Model	1-48
Simulating the Model	1-49
Create and Simulate a Model That Changes a Parameter	
Value	1-52
Overview	1-52
Opening the Example Model	1-53
Applying an Alternate Parameter Value Using a	
Variant	1-53
Simulating the Model With and Without the Variant	1-54
Create and Simulate a Model with a User-Defined	
Function	1-57
Overview	1-57
Creating a Custom Function	1-59
Opening the Example Model	1-60
Adding the Custom Function to the Example Model	1-60
Defining a Rule to Affect Parameter Value	1-61
Adding an Event to Reset the Solver at a Discontinuity	1-61
Simulating the Modified Model	1-61
See Also	1-01 1-65
Бостщобо	T-00
View Model Equations	1-66
View Model Equations	1-00

Overview of Structural Analysis	2-2
Model Verification	2-3
What is Model Verification?	2-3
When to Verify a Model	2-3
Verifying That a Model Has No Warnings or Errors	2-4
More About	2-4
Verifying a Model	2-5
Conserved Moiety Determination	2-6
Introduction to Moiety Conservation	2-6
Algorithms for Conserved Cycle Calculations	2-6
More About	2-8
Determining Conserved Moieties	2-9
Determining the Adjacency Matrix for a Model	2-13
What Is an Adjacency Matrix?	2-13
Retrieving an Adjacency Matrix for a Model	2-14
Determining the Stoichiometry Matrix for a Model	2-16
What Is a Stoichiometry Matrix?	2-16
Retrieving a Stoichiometry Matrix for a Model	$\frac{2}{2}$ -17

# Simulation and Analysis

# 3

2

Overview of Simulation and Analysis	3-2
Typical Workflow	3-2
Model Simulation	3-3
Simulating a Model	3-3

Plotting Simulation Results	3-4
Interpreting Simulation Results	3-4
Configuring Stop Time and Other Simulation Settings	3-4
Choosing a Simulation Solver	3-5
SUNDIALS Solvers	3-6
Stochastic Solvers	3-6
Ensemble Runs of Stochastic Simulations	3-11
See Also	3-11
Simulate a Model and View Results	3-12
Overview	3-12
Loading the Example Model	3-13
Configuring Simulation Settings	3-13
Simulating the Model	3-14
Plotting Simulation Results	3-14
	3-14 3-17
Extracting Data for Analysis	9-17
Sensitivity Calculation	3-18
About Calculating Sensitivities	3-18
Model Requirements for Calculating Sensitivities	3-18
Setting SolverOptions Properties for Calculating	
Sensitivities	3-20
Calculating Sensitivities of Model and Viewing Results	3-20
See Also	3-21
References	3-21
Calculate Sensitivities	3-22
Overview	3-22
Loading and Configuring the Model for Sensitivity	
Analysis	3-23
Performing Sensitivity Analysis	3 - 24
Extracting and Plotting Sensitivity Data	3-24
Perform Parameter Scan	3-27
Parameter Estimation	3-29
About Parameter Estimation	3-29
Estimating Parameters of a Model	3-29
Specifying Solver Type and Options for Parameter	
Estimation	3-30
See Also	3-30
Individual and Population Fitting	3-30

Estimate Parameters	3-32
Overview	3-32
Loading the Example Model	3-33
Defining Experimental Data	3-33
Simulating the G Protein Model	3-34
Estimating the kGd Parameter in the G Protein Model	3-36
Simulating and Plotting Results Using the Estimated	
Parameter	3-38
Estimating Other Parameters in the G Protein Model	3-39
Accelerating Model Simulations and Analyses	3-44
What Is Acceleration?	3-44
What Simulations and Analyses Can Be Accelerated?	3-44
When to Accelerate Simulations and Analyses	3-45
Prerequisites for Accelerating Simulations and	
Analyses	3-45
Accelerate a Simulation or Analysis	3-45
Troubleshooting Accelerated Simulations and Analyses	3-46

# Pharmacokinetic Modeling

# 4

Pharmacokinetic Modeling Functionality	4-2
Overview	4-2
Required and Recommended Software for Pharmacokinetic	
Modeling	4-3
How SimBiology Supports Pharmacokinetic Modeling	4-4
Using the Command Line Versus the SimBiology	
Desktop	4-5
Pharmacokinetic Modeling Example	4-6
Acknowledgements: Tobramycin Data Set	4-6
Importing Data — Supported Files and Data Types Supported Files and Data Types	4-8 4-8
Support for Importing NONMEM Formatted Files	4-8
Creating a Data File with SimBiology Definitions	4-12
Importing Data	4-13
Importing Data from Files	4-13

Importing Data from NONMEM Formatted Files            Other Resources for Importing Data	4-14 4-15
Creating Pharmacokinetic Models	4-17
Ways to Create or Import Pharmacokinetic Model How SimBiology Models Represent Pharmacokinetic	4-17
Models Creating Pharmacokinetic Model Using the Command	4-17
Line	4-19
Dosing Types	4-21
Elimination Types Intercompartmental Clearance	4-24 4-26
Unit Conversion for Imported Data	4-20 4-27
About Data Fitting in PKPD Models	4-29
Data Fitting Functionality	4-29 4-30
Prerequisites for Data Fitting Prerequisites for Using Custom SimBiology Models in Data	4-30
Fitting	4-30
Performing Data Fitting with PKPD Models	4-34
Data Fitting Workflow	4-34
Specifying and Classifying the Data to Fit	4-35
Specifying Solver Type and Options for Fitting Setting Initial Estimates	4-38 4-38
Specifying a Nonlinear, Mixed-Effects Model	4-30
Specifying a Covariate Model	4-41
Specifying the Covariance Pattern of Random Effects	4-43
Specifying an Error Model	4-45
Specifying Parameter Transformations	4-45
Performing Population Fitting	4-46
Performing Individual Fitting	4-51
PK/PD Modeling and Simulation to Guide Dosing Strategy for Antibiotics	4-54
Strategy for Antibiotics	4-94
Deploy a SimBiology Model	4-68

# A

Defining Reaction Rates with Mass Action Kinetics	A-2
Definition of Mass Action Kinetics	A-2
Zero-Order Reactions	A-2
First-Order Reactions	A-3
Second-Order Reactions	A-4
Reversible Mass Action	A-6
Defining Reaction Rates with Enzyme Kinetics	A-8
Simple Model for Single Substrate Catalyzed Reactions	A-8
Enzyme Reactions with Differential Rate Equations	A-8
Enzyme Reactions with Mass Action Kinetics	A-10
Enzyme Reactions with Irreversible	
Henri-Michaelis-Menten Kinetics	A-11

#### **Creating Rate Rules**

# B

Using Rate Rules When the Rate of Change Is Constant	B-2
Using Rate Rules When the Rate of Change Is Exponential	<b>B-</b> 4
Using Rate Rules When the Rate of Change Is Determined by Another Species	B-6
Using Rate Rules To Express Differential Rate Equations as Rules	B-8

Minimal Cascade Model for a Mitotic Oscillator	C-2
Goldbeter Model	C-2
SimBiology Model with Rate Rules	C-5
SimBiology Model with Reactions	C-7
References	C-18
Model of the Yeast Heterotrimeric G Protein Cycle	C-19
Background on G Protein Cycles	C-19
Modeling a G Protein Cycle	C-20
References	<b>C-2</b> 4
Model of M-Phase Control in Xenopus Oocyte	
Extracts	C-25
M-Phase Control Model	C-25
M-Phase Control Equations	C-27
SimBiology Model with Rate and Algebraic Rules	C-36
SimBiology Model with Reactions and Algebraic Rules	C-43
References	C-60

С

l

# Index

# Modeling

- "What is a Model?" on page 1-2
- "Model Modifiers" on page 1-6
- "Representing a Model and Model Modifiers" on page 1-7
- "Model Object" on page 1-9
- "Objects Representing Quantities" on page 1-10
- "Compartment Object" on page 1-11
- "Species Object" on page 1-12
- "Parameter Object" on page 1-16
- "Objects Representing Expressions" on page 1-17
- "Reaction Object" on page 1-18
- "Rule Object" on page 1-25
- "Event Object" on page 1-32
- "Objects Representing Model Modifiers" on page 1-42
- "Variant Object" on page 1-43
- "Dose Object" on page 1-45
- "Create and Simulate a Model That Changes a Species Amount" on page 1-47
- "Create and Simulate a Model That Changes a Parameter Value" on page 1-52
- "Create and Simulate a Model with a User-Defined Function" on page 1-57
- "View Model Equations" on page 1-66

# What is a Model?

#### In this section...

"Model Definition" on page 1-2

"Expressions" on page 1-2

"Quantities" on page 1-3

"Model Hierarchy" on page 1-4

"More About" on page 1-5

## **Model Definition**

A SimBiology<sup>®</sup> *model* is composed of a set of expressions (reactions, differential equations, discrete events), which together describe the dynamics of a biological system. You write expressions in terms of quantities (compartments, species, parameters), which are also enumerated in the model.

#### **Expressions**

There are three distinct types of expressions in SimBiology:

- Reactions
- Rules
- Events

#### Reactions

A *reaction* describes a process such as a transformation, transport, or binding/unbinding process between reactants and products.

Example reactions include:

Creatine + ATP <-> ADP + phosphocreatine cytoplasm.speciesA -> nucleus.speciesA

#### Rules

A *rule* is a class of mathematical expressions that include differential equations, initial assignments, repeated assignments, and algebraic constraints.

For example, you can use a rule to:

- Specify values for model components that are required for comparison with experimental data. For example, specify the active fraction of total protein.
- Assign values to model components based on the values of other components in the model. For example, define a parameter's value as being proportional to a species or another parameter.
- Define mass balance equations.
- For species, use rate rules as an alternative to the differential rate expression generated from reactions.

#### **Events**

An *event* describes an instantaneous change in the value of a quantity (compartment, species, parameter). The discrete transition occurs when a user-specified condition becomes true. The condition can be a specific time or a specific time-independent condition.

For example, you can use an event to:

- Activate or deactivate a specific species (activator or inhibitor species)
- Change a parameter value based on external signals
- Change reaction rates in response to addition or removal of a species
- Replicate an experimental condition, such as the addition or removal of an activating agent (such as a drug) to or from a sample

#### **Quantities**

SimBiology uses three types of quantities in models:

- Compartments
- Species

• Parameters

#### Compartments

A *compartment* defines a physically bounded region that contains species. A compartment is characterized by a capacity expressed as volume, area, or length. A compartment can also contain other compartments, which adds hierarchy to a model. For example, a compartment named cytoplasm might contain a compartment named nucleus, thereby partitioning species based on their location.

#### **Species**

A *species* characterizes the state of the biological system by representing the amount (or concentration) present in the system for that entity. Examples of species are DNA, ATP, and creatine. Species' amounts (or concentrations) vary during a simulation as a result of their participation in reactions, differential equations, and events. Therefore, species represent the dynamical state of a biological system.

#### **Parameters**

A *parameter* is a quantity that is referred to by expressions. It typically remains constant during a simulation. For example, parameters are used as rate constants in reactions.

You can configure a parameter to vary during a simulation. This is useful, for example, to model the change in a reaction rate given the concentration of a catalyst or a change in temperature.

#### **Model Hierarchy**

Note the following conditions imposed on quantities in the model hierarchy:

- Models must contain at least one compartment.
- A compartment can contain one or more compartments.
- Species are always contained within a compartment.

#### **More About**

"Representing a Model and Model Modifiers" on page 1-7

# **Model Modifiers**

#### In this section...

"Variants" on page 1-6

"Doses" on page 1-6

In addition to expressions and quantities, which are model components, SimBiology provides the following constructs (or objects) that you use to modify or perturb a model from its base configuration.

#### Variants

A *variants* is a collection of quantities (compartments, species, and/or parameters) that you can use to alter a model's initial or base configuration, which is easier than individually modifying each quantity separately. For example, assuming that a different set of parameter values characterizes differences between wild type and mutant strains, you can use a variant to group parameter values indicative of these strains. You apply variants to a model to evaluate the model behavior under "variant" conditions. Note that the model's original configuration is only temporarily altered, for example during a simulation.

For example, you can use a variant to compare:

- Two different species, such as human versus mouse
- Wild type versus mutant strains
- Different experimental conditions

#### Doses

A *dose* is used to increment the amount (or concentration) of a species exogenously. For example, you can use a dose to model the instantaneous supply of a drug regimen during the simulation of a model.

## **Representing a Model and Model Modifiers**

#### In this section...

"Constructing a Simple Model" on page 1-7

```
"SimBiology Objects" on page 1-7
```

#### **Constructing a Simple Model**

The following code shows how to construct a simple model consisting of one compartment, two species, a parameter, and a reaction:

```
% Create a model named example
model = sbiomodel('example');
% Add a compartment named cell to model
compartment = addcompartment(model, 'cell');
% Add two species, A and B, to the cell compartment
species_1 = addspecies(compartment, 'A');
species_2 = addspecies(compartment, 'B');
% Add a parameter, K1, to model with a value of 3
parameter = addparameter(model, 'K1', 3);
% Add the reaction A -> B to the model
reaction = addreaction(model, 'A -> B', 'ReactionRate', 'K1');
```

### **SimBiology Objects**

In SimBiology, models and their components are implemented as objects. For example, in the previous code, model is a model object composed of a compartment object, compartment, which in turn is composed of species, parameter and reaction objects. These objects have properties and methods associated with them, which you use to access and configure them. Use the get method to list the property values of an object. Use the set method to change the property values of an object.

SimBiology objects are handle objects, which has implications for how they behave during copy operations. In particular, handle objects do not behave as arrays of doubles do in MATLAB<sup>®</sup>. To learn how handle objects affect copy operations, see Copying Objects in the MATLAB Programming Fundamentals documentation.

1

#### **More About**

- "Model Object" on page 1-9
- "Objects Representing Quantities" on page 1-10
- "Compartment Object" on page 1-11
- "Species Object" on page 1-12
- "Parameter Object" on page 1-16
- "Objects Representing Expressions" on page 1-17
- "Reaction Object" on page 1-18
- "Rule Object" on page 1-25
- "Event Object" on page 1-32
- "Objects Representing Model Modifiers" on page 1-42
- "Variant Object" on page 1-43
- "Dose Object" on page 1-45

# **Model Object**

A model object represents a model and is composed of quantities and expressions. Quantities represent the state variables in the system while expressions depict the relationships between quantities and therefore describe the dynamics of the model.

For information about	See
Creating a model	sbiomodel
Methods and properties of a model	model object
Removing models from MATLAB Workspace	clear
Deleting models	sbioreset

# **Objects Representing Quantities**

The following objects represent quantities in a model:

- Compartment
- Species
- Parameter

#### Scoping of Compartments, Species, and Parameters

Scoping refers to which object another object is contained in. Scoping affects compartments, species, and parameters.

- **Compartment Object** A compartment is scoped to (or contained in) a model or another compartment.
- **Species Object** Although a model can contain multiple compartments, each species is scoped to (or contained in) only one compartment.
- **Parameter Object** A parameter is scoped to (or contained in) a model or a kinetic law.

#### **Naming of Compartments and Species**

Note the following when naming objects within a model:

- Compartment names must be unique within a model.
- ٠

#### **More About**

- "Compartment Object" on page 1-11
- "Species Object" on page 1-12
- "Parameter Object" on page 1-16

## **Compartment Object**

A compartment object represents a compartment, which is a physically isolated region. It lets you associate pools of species to that physically isolated region. It has a capacity associated with it.

All models must contain at least one compartment. A compartment is scoped to a model or another compartment. A compartment contains one or more species. Each compartment within a model must have a unique name.

You can add a compartment explicitly (using the addcompartment method) or add a reaction (using the addreaction method) to create a compartment.

For information about	See
Creating and adding a compartment to a model	addcompartment, addreaction
Methods and properties of a compartment	compartment object

# **Species Object**

A species object represents a species, which is the amount of a chemical or entity that participates in reactions. A species is always scoped to a compartment. Each species must

When adding species to a model with multiple compartments, you must specify qualified names, using *compartmentName.speciesName*. For example, nucleus.DNA denotes the species DNA in the compartment nucleus.

For information about	See
Creating and adding a species to a model	addspecies
Methods and properties of a species	species object

#### How Species Amounts Change During Simulations

The amount of a species can remain constant or vary during the simulation of a model. Use the following properties of a **species object** to specify how the amount of a species changes during a simulation:

- ConstantAmount property When set to true, the species amount does not change during a simulation. The species can be part of a reaction or rule, but the reaction or rule cannot change its amount. When set to false, the species amount is determined by a reaction or a rule, but not both.
- BoundaryCondition property When set to true, the species amount is either constant or determined by a rule, but not determined by a chemical reaction. In other words, the simulation does not create a differential rate term from the reactions for this species, even if it is in a reaction, but it can have a differential rate term created from a rule.

#### **Keeping a Species Amount Unchanged**

Set ConstantAmount to true and BoundaryCondition to false for a constant species, whose amount is not changed by a reaction or rule. In this case, the species acts like a parameter. It cannot be in a reaction, and it cannot be varied by a rule.

ConstantAmount	BoundaryCondition	Reaction	Rule	Changed By
True	False	No	No	Never

**Example** — Species **E** is not part of the reaction, but it is part of the reaction rate equation. **E** is constant and could be replaced with the constant Vm = k2\*E.

```
reaction: S -> P
reaction rate: kcat*E*S/(Km + S)
```

#### Changing a Species Amount with a Reaction or Rule

Set ConstantAmount to false and BoundaryCondition to false for a species whose amount is changed by a reaction or rule, but not both.

ConstantAmount	BoundaryCondition	Reaction	Rule	Changed By
False	False	Yes	No	Reaction
False	False	No	Yes	Rule

**Example 1** — Species A is part of a reaction, and it is in the reaction rate equation. The species amount or concentration is determined by the reaction. This is the most common category of a species. A differential rate equation for the species is created from the reactions.

```
reaction: A -> B
reaction rate: k*A
```

**Example 2** — Species  $\mathbf{E}$  is not part of the reaction, but it is in the reaction rate equation.  $\mathbf{E}$  varies with another reaction or rule.

```
reaction: S -> P
reaction rate: kcat*E*S/(Km + S)
```

**Example 3** — Species **G** is not part of a reaction, and it is not in a rate equation. **G** varies with an algebraic rule or rate rule.

rate rule: dG/dt = k

#### Changing a Species Amount with a Rule When Species is Part of a Reaction

Set ConstantAmount to false and BoundaryCondition to true for a species whose amount is changed by a rule, but the species is also part of a reaction, and a differential rate term from the reaction is not created. The amount of the species changes with the rule, and a differential rate term is created from the rule.

ConstantAmount	BoundaryCondition	Reaction	Rule	Changed By
False	True	Yes	Yes	Rule

**Example 1** — Species A is not changed by the rate equation, but changes according to a rate rule. However, A could be in the rate equation that changes other species in the reaction.

```
reaction: A -> B
reaction rate: k1 or k1*A
rate rule: dA/dt = k2*A (solution is A = k2*t)
        (enter in SimBiology as A = k2*A)
```

**Example 2** — Species A is not in the rate equation, but changes according to an algebraic rule.

# Keeping a Species Amount Unchanged When Species is Part of a Reaction that Adds or Removes Mass

Set ConstantAmount to false and BoundaryCondition to true for a constant species that is part of a reaction, but a differential rate term is not created from the reaction. The differential rate term is created from a rule.

ConstantAmount	BoundaryCondition	Reaction	Rule	Changed By
True	True	Yes	No	Never

During simulation, a differential rate equation is not created for the species. dSpecies/dt does not exist.

**Example 1**—A is a *infinite source* and its amount does not change. B increases with a zero order rate (k and k\*A are both constants). A source refers to a species where mass is added to the system.

reaction: A -> B reaction rate: k or k\*A

**Example 2** — B decreases with a first-order rate, but **A** is an *infinite sink* and its amount does not change. A *sink* refers to a species where mass is subtracted from the system.

reaction: B -> A
reaction rate: k\*B

**Example 3** — The **null** species is a reserved species name that can act as a source or a sink.

```
reaction: null -> B
reaction rate: k
reaction: B -> null
reaction rate: k*B
```

**Example 4** — **ATP** and **ADP** are in the reaction and have constant values, but they are not in the reaction rate equation.

reaction: S + ATP -> P + ADP
reaction rate: Vm\*S/(Km + S)

# **Parameter Object**

A parameter object represents a parameter, which is a value that typically remains constant during a simulation. For example, you use parameters to define reaction rate constants. In some circumstances it is useful to allow parameter values to vary. In these cases you can specify a parameter as nonconstant.

For information about	See
Creating and adding a parameter to a model	addparameter
Methods and properties of a parameter	parameter object

#### **Scope of Parameter Objects**

When you create a parameter, you scope it to either a model or a reaction.

#### Parameters Scoped to a Model

Parameters scoped to a model can be used (or referenced) by any expression (reaction, rule, or event) in the model.

#### **Parameters Scoped to a Reaction**

Parameters scoped to a reaction can be used (or referenced) by only the reaction rate expression.

# **Objects Representing Expressions**

The following objects represent expressions in a model:

- Reaction object
- KineticLaw object
- Rule object
- Event object

# When Reactions, Rules, and Events Specify Parameters

Reactions, rules and events can specify one or more parameters. A parameter is scoped a model or a kinetic law. Note the following when using a reaction, rule, or event to specify a parameter:

- When a *reaction* specifies a parameter, the parameter can be scoped to the model or the kinetic law that is part of that reaction. If more than one reaction specifies the same parameter, the parameter must be scoped to the model. If two parameters have the same name, one at the model level and the other at the kinetic law level, the software uses the parameter at the kinetic law level for the reaction rate that specifies the parameter.
- When a *rule* specifies a parameter, the parameter must be scoped to the model.
- When an *event* specifies a parameter, the parameter must be scoped to the model.

For more information, see "Scope of Parameter Objects" on page 1-16.

#### **More About**

- "Reaction Object" on page 1-18
- "Rule Object" on page 1-25
- "Event Object" on page 1-32
- "Create and Simulate a Model with a User-Defined Function" on page 1-57

# **Reaction Object**

A reaction object represents a reaction, which is a mathematical expression and other information that describe a transformation, transport, or binding process that changes one or more species. Typically, the change is to the amount of a species.

A reaction object includes:

- Reaction property Mathematical expression that describes the reaction
- ReactionRate property Mathematical expression that defines the rate at which the reactants combine to form products. You can provide this information explicitly or use the KineticLaw property to populate this information.
- KineticLaw property Object that specifies a rate law that defines the type of reaction rate. Examples include Henri-Michaelis-Menten and Mass Action. The object also specifies species objects, or parameter objects. This property is optional. It serves as a template for a reaction rate and provides a convenient way of applying a specific rate law to multiple reactions. If you use this property, it automatically populates the ReactionRate property.

For information about	See
Creating and adding a reaction to a model	addreaction
Methods and properties of a reaction	reaction object
Creating and adding a kinetic law to a reaction	addkineticlaw
Methods and properties of a kinetic law	KineticLaw object

A reaction is scoped to a model.

#### Writing Reaction Expressions

Use standard chemistry reaction notation to create the mathematical expression for a reaction (Reaction property of a reaction object).

Following are rules for writing reaction expressions:

- Use spaces before and after species names and stoichiometric values.
- Stoichiometry values must be positive.
- If a stoichiometry value is not specified, it is assumed to be 1.
- In a model with a single compartment, specify species using *speciesName*. In a model with multiple compartments, specify species using qualified names: *compartmentName.speciesName*. For example, nucleus.DNA denotes the species DNA in the compartment nucleus.
- Enclose names with non-alphanumeric characters (including spaces) in brackets.
- Reactions can be reversible (<->) or irreversible (->).

Examples of reaction expressions include:

```
Creatine + ATP <-> ADP + phosphocreatine
glucose + 2 ADP + 2 Pi -> 2 lactic acid + 2 ATP + 2 H2O
cytoplasm.A -> nucleus.A
[compartment 1].[species A] -> [compartment 2].[species A]
```

#### Writing Reaction Rate Expressions Explicitly

Use any valid MATLAB code to create the mathematical expression for a reaction rate (ReactionRate property of a reaction object). The reaction rate can specify compartments, species, or parameters.

Following are rules for writing reaction rate expressions:

- The expression must be a single MATLAB statement that returns a scalar.
- In a model with a single compartment, specify species using *speciesName*. In a model with multiple compartments, specify species using qualified names: *compartmentName.speciesName*. For example, nucleus.DNA denotes the species DNA in the compartment nucleus.

- Enclose names with non-alphanumeric characters (including spaces) in brackets.
- Do not end the reaction rate expression with any of the following:
  - Semicolon
  - Comma
  - Comment text preceded by %
  - Line continuations indicated by ...

For example, if you have the following reaction expression:

Creatine + ATP <-> ADP + phosphocreatine

and the reaction follows Mass Action kinetics, then the reaction rate expression would be:

```
K*Creatine*ATP - Krev*ADP*phosphocreatine
```

**Tip** If your reaction rate expression is not continuous and differentiable, see "Using Events to Address Discontinuities in Rule and Reaction Rate Expressions" on page 1-40 before simulating your model.

#### Creating Reaction Rate Expressions Using Kinetic Law Objects

A KineticLaw object is scoped to a reaction and specifies:

- A rate law that defines the type of reaction rate. Examples include Henri-Michaelis-Menten and Mass Action.
- species and parameters

A KineticLaw object serves as a template for a reaction rate and provides a convenient way of applying a specific rate law to multiple reactions. You can use this object to create a reaction rate, which populates the ReactionRate property of the reaction object.

For example, if you create a KineticLaw object that specifies Henri-Michaelis-Menten for the KineticLawName, species S, and parameters Vm and Km, the reaction rate law is:

$$V_{\rm m} * S / (K_m + S)$$

Then if you create a reaction object that specifies the previous KineticLaw object and species the following reaction expression:

A -> B

with Vm = Va and Km = Ka and S = A, then the reaction rate equation is:

Va\*A/(Ka + A)

#### **Examples of Creating Reaction Rates**

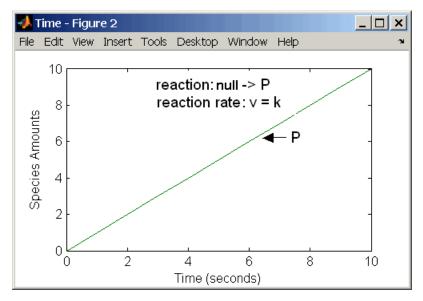
#### **Example of Creating a Zero-Order Reaction**

With a zero-order reaction, the reaction rate does not depend on the concentration of reactants. Examples of zero-order reactions are synthesis from a null species, and modeling a source species that is added to the system at a specified rate.

```
reaction: null -> P
reaction rate: k mole/second
    species: P = 0 mole
    parameters: k = 1 mole/second
```

**Note** When specifying a null species, the reaction rate must be defined in units of amount per unit time not concentration per unit time.

Entering the reaction above into the software and simulating produces the following result:



**Zero-Order Mass Action Kinetics** 

**Note** If the amount of a reactant with zero-order kinetics reaches zero before the end of a simulation, then the amount of reactant can go below zero regardless of the solver or tolerances you set.

#### **Examples of Creating Other Reactions**

For examples of creating other reaction rates, see the following sections in Appendix A, "Creating Reaction Rates".

- "Defining Reaction Rates with Mass Action Kinetics" on page A-2
- "Defining Reaction Rates with Enzyme Kinetics" on page A-8

#### How Reaction Rates Are Evaluated

#### **Reaction Rate Dimensions**

When calculating species fluxes, SimBiology must determine whether you specified reaction rates in dimensions of amount/time or concentration/time.

When all compartments in a model have a capacity of one unit, amount and concentration are numerically equivalent.

For all other models, the numerical results of the simulation depend on which interpretation SimBiology selects. SimBiology determines whether a reaction rate is in dimensions of amount/time or concentration/time via dimensional analysis of ReactionRate expressions. This minimum level of dimensional analysis always occurs, even when DimensionalAnalysis and UnitConversion are off.

The DefaultSpeciesDimension property defines the dimensions of species appearing in a reaction rate. SimBiology infers the dimensions of parameters appearing in a reaction rate from their ValueUnits property. If any parameters appearing in a reaction rate expression do not have units, SimBiology interprets the reaction rate in dimensions of amount/time. Therefore, the only way to specify that a reaction rate has dimensions of concentration/time is to assign appropriate units to all parameters.

#### **Reactions Spanning Multiple Compartments**

Specify reactions that span compartments using the syntax compartment1Name.species1Name -> compartment2Name.species2Name. The reaction rate dimensions must resolve to amount/time when:

- Species span multiple compartments.
- The reaction is reversible mass action and the products are in multiple compartments.

#### **Examples**

Consider a reaction a + b > c. Using mass action kinetics, the reaction rate is k\*a\*b, where k is the rate constant of the reaction. If you specify that initial amounts of a and b are 0.01 molarity and 0.005 molarity respectively, then the reaction rate is in concentration/time (and units of molarity/second) if the units of k are 1/(molarity\*second). If you specify k with another equivalent unit definition, for example, 1/((moles/liter)\*second), SimBiology checks whether the physical quantities match. If the physical quantities do not match, you see an error and the model is not simulated. If, in the previous example, you specify that initial amounts of a and b are 0.01 and 0.005 respectively, without specifying units, SimBiology checks whether DefaultSpeciesDimension is substance or concentration. If DefaultSpeciesDimension is concentration, and you set units on the rate constant such that the reaction rate dimensions resolve to concentration/time, SimBiology scales the species amounts for compartment capacity, and returns the species values in concentration.

If you specify initial amounts of a and b as 0.01 molarity and 0.005 mole respectively, include the volume scaling for b in the reaction rate expression. Include volume scaling in the rate constant, and set the units of the rate constant accordingly (1/(mole\*second) for concentration/time, or 1/(molarity\*second) for amount/time).

#### **Viewing Equations for Reactions**

You can view the system of equations that SimBiology creates when you build a model using reaction expressions. For details, see "View Model Equations" on page 1-66.

#### **More About**

"Create and Simulate a Model with a User-Defined Function" on page 1-57

## **Rule Object**

A rule object represents a rule, which is a mathematical expression that modifies one of the following:

- Compartment capacity
- Species amount
- Parameter value

A rule is scoped to a model. A rule has a RuleType property that specifies one of the following four types of rules:

- Algebraic
- Initial Assignment
- Repeated Assignment
- Rate

For information about	See
Creating and adding a rule to a model	addrule
Methods and properties of a rule	rule object

#### Writing Rule Expressions

Use any valid MATLAB code to create the mathematical expression for a rule. The rule can specify compartments, species, or model-scoped parameters.

Following are rules for writing rule expressions:

- The expression must be a single MATLAB statement.
- Do not end the rule expression with any of the following:
  - Semicolon
  - Comma
  - Comment text preceded by %

Line continuations indicated by ...

#### Tip

- If you plan to specify a parameter in a rule, set the scope of the parameter to the model.
- If you use an algebraic or rate rule to vary the value of a parameter during the simulation, make sure the ConstantValue property of the parameter is set to false.
- If your algebraic, repeated assignment, or rate rule expression is not continuous and differentiable, see "Using Events to Address Discontinuities in Rule and Reaction Rate Expressions" on page 1-40 before simulating your model.

#### **Algebraic Rules**

An algebraic rule lets you specify mathematical constraints on one or more compartments, species, or parameters that must hold during a simulation. It is evaluated continuously during a simulation.

An algebraic rule takes the form 0 = Expression and the rule is specified as the Expression.

For example, you could write a mass conservation expression such as species\_total = species1 + species2 where species\_total is the independent variable. In SimBiology, write the rule as species1 + species2 - species\_total.

An algebraic rule is a convenient way to define mathematical relationships between states. A model can consist of a combination of differential and algebraic relationships.

An algebraic rule is a constraint that is enforced by the solver during simulation. You can use algebraic rules to specify the dynamics for parameters, species, and compartments that are not driven by one or more reactions. The accuracy of the solution depends on the tolerance specified in the Configset object, which defines the simulation settings. An algebraic rule is defined by the equation:

f(t,x) = 0

Where t is simulation time. The variable x is species amount, parameter value, or compartment capacity.

An example of an algebraic rule is:

x\*log(x) - 3

#### **Considerations When Imposing Constraints**

Consider the mathematical constraint y = m\*x - c. In the software this rule is written as m\*x - c - y. If you want to use this rule to determine the value of y, then m, x, and c must be variables or constants whose values are known or determined by other equations. In general, the degree of freedom available must match the number of constraints. Therefore, you must ensure that the equation is not overconstrained or underconstrained. In this example, if the equation is underconstrained, it is unclear which variable is being determined by the expression.

#### **Initial Assignment Rules**

An initial assignment rule lets you specify the initial value of a compartment capacity, species amount, or parameter value as a function of other component values in the model. It is evaluated once at the beginning of a simulation.

An initial assignment rule is expressed as Variable = Expression.

For example, you could write an initial assignment rule to set the initial amount of species1 to be proportional to species2:

species1 = k\*species2

#### **Repeated Assignment Rules**

A repeated assignment rule lets you specify a value that holds at all times during simulation, and is a function of other component values in the model. It is evaluated at every time-step during a simulation. These time steps are determined by the solver during the simulation process. A repeated assignment rule is expressed as Variable = Expression.

For example, if you want the capacity of a compartment (cytoplasm) to change in response to a change in the concentration of a species (x), write a repeated assignment rule to set the capacity of cytoplasm to be proportional to x.

cytoplasm = k\*x

Where k is a specified constant parameter.

#### **Repeated Assignment Versus Algebraic Rules**

Repeated assignment rules are mathematically equivalent to algebraic rules, but result in exact solutions, compared to algebraic rules whose accuracy depends on the tolerance specified in the Configset object, which defines the simulation settings.

#### Tip

- If you can solve for the variable, use a repeated assignment rule instead of an algebraic rule.
- In repeated assignment rules, the constrained variable is explicitly defined as the left-hand side, whereas in algebraic rules it is inferred from the degrees of freedom in the system of equations. See also "Considerations When Imposing Constraints" on page 1-27.

#### **Rate Rules**

A rate rule lets you specify the time derivative of a compartment capacity, species amount, or parameter value. It is evaluated continuously during a simulation.

A rate rule is determined by dVariable/dt = Expression, which is expressed in the software as Variable = Expression. For example, to define the rate of change in the quantity of species3 (dspecies3/dt), write the rule in the software as:

```
species3 = k * (species1 + species2)
```

One example of a rate rule is when species1 is at the boundary of the system, but the rate of input of species1 to the system can be determined by a rate rule.

A rate rule is defined by the equation:

dx/dt = f(t,W)

The variable x can be a species amount, parameter value, or compartment capacity. The function f(W) is an expression that can include other species and parameters. Enter a rate rule using the form

x = f(t, W)

#### **Examples of Creating Rate Rules**

## Example of Creating a Rate Rule When the Rate of Change Is Constant

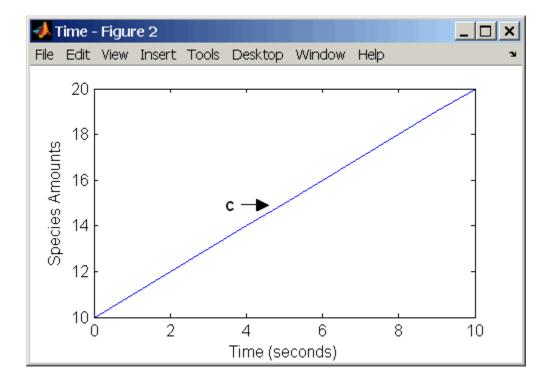
You can increase or decrease the amount or concentration of a species by a constant value using a zero-order rate rule. For example, suppose species c increases by a constant rate k.

```
reaction: none
rate equation: none
rate rule: dc/dt = k
species : c = 10 mole(initial amount)
parameters: k = 1 mole/second
```

The analytical solution is  $c = kt + c_0$ , where  $c_0$  is the initial amount or concentration of the species c.

Enter the rule described above as c = k. Set the RuleType property to rate, enter the values for c and k, and then simulate.

1



#### **Examples of Creating Other Rate Rules**

For examples of creating other rate rules, see the following sections in Appendix B, "Creating Rate Rules":

- "Using Rate Rules When the Rate of Change Is Exponential" on page B-4
- "Using Rate Rules When the Rate of Change Is Determined by Another Species" on page B-6
- "Using Rate Rules To Express Differential Rate Equations as Rules" on page B-8

## **More About**

For an example of creating a model that includes a rule expression that calls the custom function, see "Create and Simulate a Model with a User-Defined Function" on page 1-57.

## **Event Object**

An event object represents an event, which is a discrete transition in value of a quantity or expression in a model. This discrete transition occurs when a user-specified condition becomes true. The condition can be a specific time or a time-independent condition.

An event is scoped to a model.

For information about	See
Creating and adding an event to a model	addevent
Methods and properties of a event	event object

For an example of creating and using an event in a model, see "Create and Simulate a Model That Changes a Species Amount" on page 1-47.

#### **Event Triggers**

An event has a Trigger property that specifies a condition that must be true to trigger the event to execute.

Typical event triggers are:

- A specific time during simulation Specify that the event must change the amounts or values of species or parameters. For example, at time = 5 s, increase the amount of an inhibitor species above the threshold to inhibit a given reaction.
- In response to state or changes in the system Change amounts/values of certain species/parameters in response to events that are not tied to any specific time. For example, when species A reaches an amount of 30 molecules, double the value of reaction rate constant k; or when temperature reaches 42 C, inhibit a particular reaction by setting its reaction rate to zero.

## **Event Functions**

An event has an EventFcns property that specifies what occurs when the event is triggered. Event functions can range from simple to complex. For example, an event function might:

- Change the values of compartments, species, or parameters.
- Double the value of a reaction rate constant.

## **Specifying Event Triggers**

The Trigger property of an event specifies a condition that must become true for an event to execute. Typically, the condition uses a combination of relational and logical operators to build a trigger expression.

A trigger can contain the keyword time and relational operators to trigger an event that occurs at a specific time during the simulation. For example, time  $\geq x$ . For more information see the Trigger property reference page.

MATLAB uses specific operator precedence to evaluate trigger expressions. Precedence levels determine the order in which MATLAB evaluates an expression. Within each precedence level, operators have equal precedence and are evaluated from left to right. To find more information on how relational and logical operators are evaluated see "Operators" in the MATLAB Programming Fundamentals documentation.

Following are rules for writing expressions for event triggers:

- The expression must be a single MATLAB statement that returns a logical.
- Do not end the expression with any of the following:
  - Semicolon
  - Comma
  - Comment text preceded by %
  - Line continuations indicated by ...

Some examples of triggers are:

Trigger	Explanation
'(time >=5) && (speciesA<1000)'	Execute the event when the following condition becomes true:Time is greater than or equal to 5, and speciesA is less than 1000.
	<b>Tip</b> Using a && (instead of &) tells the software to evaluate the first part of the expression for whether the statement is true or false, and skip evaluating the second statement if this statement is false.
'(time >=5)    (speciesA<1000)'	Execute the event when the following condition becomes true: Time is greater than or equal to 5, or if speciesA is less than 1000.
'(s1 >=10.0)    (time>= 250) && (s2<5.0E17)'	Execute the event when the following condition becomes true: Species, s1 is greater than or equal to 10.0 or, time is greater than or equal to 250 and species s2 is less than 5.0E17.
	Because of operator precedence the expression is treated as if it were '(s1 >=10.0)    ((time>= 250) && (s2<5.0E17))'
	Thus, it is always a good idea to use parenthesis to explicitly specify the intended precedence of the statements.

Trigger	Explanation
'((s1 >=10.0)    (time>= 250)) && (s2<5.0E17)'	Execute the event when the time the following condition becomes true: Species, s1 is greater than or equal to 10 or time is greater than or equal to 250, and species s2 is less than 5.0E17.
'((s1 >=5000.0) && (time>= 250))    (s2<5.0E17)'	Execute the event when the time the following condition becomes true: Species, s1 is greater than or equal to 5000 and time is greater than or equal to 250, or species s2 is less than 5.0E17.

## **Specifying Event Functions**

The EventFcns property of an event specifies what occurs when the event is triggered. You can use an event function to change the value of a compartment, species, or parameter, or you can specify complex tasks by calling a user-defined function or script.

Following are rules for writing expressions for event functions:

- The expression must be a single MATLAB assignment statement that includes =, or a cell array of such statements.
- Do not end the statements with any of the following:
  - Semicolon
  - Comma
  - Comment text preceded by %
  - Line continuations indicated by ...

Some examples of event functions include:

1

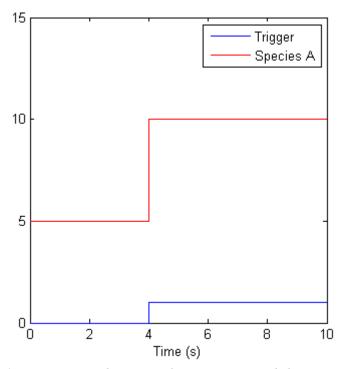
EventFcn	Explanation
'speciesA = speciesB'	When the event is executed set the amount of speciesA equal to that of speciesB.
'k = k/2'	When the event is executed halve the value of the rate constant k.
<pre>{'speciesA = speciesB', 'k = k/2'}</pre>	When the event is executed set the amount of speciesA equal to that of speciesB, and halve the value of the rate constant k.
'kC = my_func(A, B, kC)'	When the event is executed call the user-defined function my_func(). This function takes 3 arguments: The first two arguments are the current amounts of two species (A and B) during simulation and the third argument is the current value of a parameter, kC. The function returns the modified value of kC as its output.

#### **Simulation Solvers for Models Containing Events**

To simulate models containing events, use a deterministic (ODE or SUNDIALS) solver or the stochastic ssa solver. Other stochastic solvers do not support events. For more information, see "Choosing a Simulation Solver" on page 3-5.

#### How Events Are Evaluated

Consider the example of a simple event where you specify that at 4s, you want to assign a value of 10 to species A.



At time = 4 s the trigger becomes true and the event executes. In the figure above assuming that 0 is false and 1 is true, when the trigger becomes true, the amount of species A is set to 010. In theory, with a perfect solver, the event would be executed exactly at time = 4.00 s. In practice there is a very minute delay (for example you might notice that the event is executed at time = 4.00001 s). Thus, you must specify that the trigger can become true at or after 4s, which is time >= 4 s.

Trigger	EventFcn
time >= 4	A = 10

The point at which the trigger becomes true is called a *rising edge*. SimBiology events execute the EventFcn *only* at rising edges.

The Trigger is evaluated at every time step to check whether the condition specified in the trigger transitions from false to true. The solver detects and tracks *falling edges*, which is when the trigger becomes false, so if another

rising edge is encountered, the event is executed again. If a trigger is already true before a simulation starts, then the event does not execute the at the start of the simulation. The event is not executed until the solver encounters a rising edge. Very rarely, the solver might miss a rising edge; one example of this is when a rising edge follows very quickly after a falling edge, and the step size results in the solver skipping over the transition point.

If the trigger becomes true exactly at the stop time of the simulation, the event may or may not execute. If you want the event to execute, increase the stop time.

**Note** Since the rising edge is instantaneous and changes the system state, there are two values for the state at the same time. The simulation data thus contains the state before the event and after the event, but both points are at the same time value. This leads to multiple values of the system state at a single instant in time.

#### **Evaluation of Simultaneous Events**

When two or more trigger conditions simultaneously become true, the solver executes the events sequentially in the order in which they are listed in the model. You can reorder events using the reorder method. For example, consider a case where:

Event Number	Trigger	EventFcn
1	SpeciesA >= 4	SpeciesB = 10
2	SpeciesC >= 15	SpeciesB = 25

The solver tries to find the rising edge for these events within a certain level of tolerance. If this results in the two events occurring simultaneously, then the value of SpeciesB after the time step in which these two events occur, will be 25. If you reorder the events to reverse the event order, then the value of SpeciesB after the time step in which these two events occur, will be 10.

Consider an example in which you include event functions that change model components in a dependent fashion. For example, the event function in Event 2 below, stipulates that SpeciesB takes the value of SpeciesC.

Event Number	Trigger	EventFcn
1	SpeciesA >= 4	SpeciesC = 10
2	time >= 15	SpeciesB = SpeciesC

Event 1 and Event 2 may or may not occur simultaneously.

- If Event 1 and Event 2 do not occur simultaneously, when Event 2 is triggered, SpeciesB is assigned the value that SpeciesC has at the time of the event trigger.
- If Event 1 and Event 2 occur simultaneously, the solver executes Event 1 first, then executes Event 2. In the above example, if SpeciesC = 15 when the events are triggered, after the events are executed, SpeciesC = 10 and SpeciesB = 10.

## **Evaluation of Multiple Event Functions**

Consider an event function in which you specify that the value of a model component (SpeciesB) is dependent on the value of model component (SpeciesA), but SpeciesA also is changed by the event function.

Trigger	EventFcn
time >= 4	{'SpeciesA = 10, SpeciesB = SpeciesA'}

The solver stores the value of SpeciesA at the rising edge and before any event functions are executed and uses this stored value to assign SpeciesB its value. So in the above example if SpeciesA = 15 at the time the event is triggered, after the event is executed, SpeciesA = 10 and SpeciesB = 15.

## When One Event Triggers Another Event

In the example below, Event 1 includes an expression in the event function that causes Event 2 to be triggered, (assuming that SpeciesA has amount less than 5 when Event 1 is executed).

Event Number	Trigger	EventFcn
1	time >= 5	{'SpeciesA = 10, SpeciesB = 5'}
2	SpeciesA >= 5	SpeciesC = SpeciesB

When Event 1 is triggered, the solver evaluates and executes Event 1 with the result that SpeciesA = 10, and SpeciesB = 5. Now, the trigger for Event 2 becomes true (assuming that SpeciesA is below 5) and the solver executes the event function for Event 2. Thus, SpeciesC = 5 at the end of this event execution.

You can thus have event cascades of arbitrary length, for example, Event 1 triggers Event 2, which in turn triggers Event 3, and so on.

## **Cyclical Events**

In some situations, a series of events can trigger a cascade that becomes cyclical. Once you trigger a cyclical set of events, the only way to stop the simulation is by pressing **Ctrl+C**. You lose any data acquired in the current simulation. An example of cyclical events is shown below. This example assumes that **Species B** <= 4 at the start of the cycle.

Event Number	Trigger	EventFcn
1	SpeciesA > 10	{SpeciesB = 5, SpeciesC = 1'}
2	SpeciesB > 4	{SpeciesC = 10, SpeciesA = 1'}
3	SpeciesC > 9	{SpeciesA = 15, SpeciesB = 1'}

## Using Events to Address Discontinuities in Rule and Reaction Rate Expressions

The solvers provided with the SimBiology software will give inaccurate results when the following expressions are not continuous and differentiable:

- Repeated assignment rule
- Algebraic rule

- Rate rule
- Reaction rate

Either ensure that the previous expressions are continuous and differentiable or use events to reset the solver at the discontinuity, as described in Deterministic Simulation of a Model Containing a Discontinuity. 1

## **Objects Representing Model Modifiers**

Variant and dose objects can modify or perturb a model from its base configuration.

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## **More About**

- "Variant Object" on page 1-43
- "Dose Object" on page 1-45

## Variant Object

A variant object represents a variant, which is an alternate value for a compartment, species, or parameter in a model. You can apply this alternate value during a simulation, which lets you evaluate model behavior with a different value, without having to search and replace the value, or create an additional model with the new value.

You can use a variant to store an alternate value for any of the following:

- Compartment Capacity property
- Species InitialAmount property
- Parameter Value property

The alternate value applies temporarily, only during a simulation, and does not alter the model's values permanently. If you determine that the values in a variant accurately define your model, you can permanently replace the values in your model with the values stored in the variant object by using the commit method.

## **Creating and Simulating with Variants**

- 1 Create a variant object and add it to a model using the addvariant method.
- **2** (Optional) Set the Active property of the variant object to true if you always want the variant to be applied before simulating the model.
- **3** Enter the model and variant object as input arguments to sbiosimulate. This applies the variant only for the current simulation and supersedes any active variant objects on the model.

or

If you followed step 2, simply call sbiosimulate on the model object to apply the variant.

For an example of creating and using a variant in a model, see "Create and Simulate a Model That Changes a Parameter Value" on page 1-52.

For information about	See
Creating and adding a variant to a model	addvariant
Creating a stand-alone variant	sbiovariant
Methods and properties of a variant	Variant object
Appending contents to variants	addcontent
Replacing model values permanently with values from a variant	commit

## Simulating with Multiple Variants in a Model

When you use multiple variants during a simulation, and there are duplicate specifications for a property's value, the last occurrence for the property value in the array of variants is used during simulation. You can find out which variant is applied last by looking at the indices of the variant objects stored on the model.

If you specify variants as arguments to sbiosimulate, this applies the variants for the current simulation in the order that they are specified, and supersedes any active variant objects on the model.

Similarly, in the variant contents (Content property), if there are duplicate specifications for a property's value, the last occurrence for the property in the contents is used during simulation.

## **Dose Object**

A dose object represents one or more increases to the amount of a species during a model simulation. It lets you increase the amount of a species during a simulation, either at specific times or time intervals. The increase in the amount of a species occurs only during a simulation, and does not alter the species' value permanently.

For example, you can use a dose to represent the addition of a drug to a model every two hours.

There are two types of dose objects:

- ScheduleDose object Applies a dose to a single species at a pre-defined list of times.
- RepeatDose object Repeatedly applies a dose to a single species at regularly spaced time intervals.

Repeat doses and schedule doses support three kinds of increases to species amounts:

- Instantaneous increase (or step change) in the amount of a species
- Increase at a fixed rate over a period of time calculated from the dose amount
- Increase at a fixed rate calculated from the dose amount and dose duration

By using repeat doses and schedule doses with SimBiology models, you can easily model common dosing strategies, such as bolus, infusion, zero-order absorption, and first-order absorption.

#### **Creating and Simulating Dose Objects**

- 1 Create a dose and add it to the model using the adddose method.
- 2 Configure the properties of the dose. For example, set the TargetName property to the name of the species in the model that will receive the dose. Set the Active property to true to use it during a simulation.

1

**3** Enter the model as an input argument to sbiosimulate to apply the dose.

SimBiology will simulate the model using all active doses associated with the model.

For information about	See
Creating and adding a dose to a model	adddose
Creating a stand-alone dose	sbiodose
Methods and properties of a dose	ScheduleDose object and RepeatDose object

## **Simulation Solvers for Models Containing Doses**

To simulate models containing doses, use a deterministic (ODE or SUNDIALS) solver. Stochastic solvers do not support doses. For more information, see "Choosing a Simulation Solver" on page 3-5.

## Create and Simulate a Model That Changes a Species Amount

#### In this section...

"Overview" on page 1-47

"Opening the Example Model" on page 1-48

"Adding an Event to the Example Model" on page 1-48

"Simulating the Model" on page 1-49

## Overview

#### About the Example Model

This example uses the model described in Model of the Yeast Heterotrimeric G Protein Cycle.

This table shows the reactions used to model the G protein cycle and the corresponding rate parameters (rate constants) for each reaction. For reversible reactions, the forward rate parameter is listed first.

No.	Name	Reaction <sup>1</sup>	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1

No.	Name	Reaction <sup>1</sup>	Rate Parameters
6	G protein inactivation	Ga -> Gd	kGd

<sup>1</sup> Legend of species: L = ligand (alpha factor), R = alpha-factor receptor, Gd = inactive G-alpha-GDP, Gbg = free levels of G-beta:G-gamma complex, G = inactive Gbg:Gd complex, Ga = active G-alpha-GTP

#### About the Example

This example shows how to add an event to a model to trigger a time-based change. The event modifies the amount of ligand (L), thus modeling a delay in the addition of  $\alpha$ -factor to the cell culture.

For information on events and how they are evaluated, see "Event Object" on page 1-32.

## **Opening the Example Model**

Load the gprotein example project, which includes the variable m1, a model object:

```
sbioloadproject gprotein
```

The m1 model object appears in the MATLAB Workspace.

#### Adding an Event to the Example Model

 $\mbox{l}$  Set the InitialAmount of species L (ligand) to be 0.0 when the simulation starts:

```
speciesObj = sbioselect(m1, 'Type', 'species', 'Name', 'L');
set(speciesObj, 'InitialAmount', 0);
```

2 Add an event to the m1 model object. Configure the event to set the amount of species L (ligand) and to trigger when the simulation time equals 100:

evt = addevent(m1, 'time >= 100', 'L = 6.022E17');

## Simulating the Model

1 Configure the simulation settings (configset object) for the m1 model object to log all states during the simulation:

```
cs = getconfigset(m1);
set(cs.RuntimeOptions, 'StatesToLog', 'all');
```

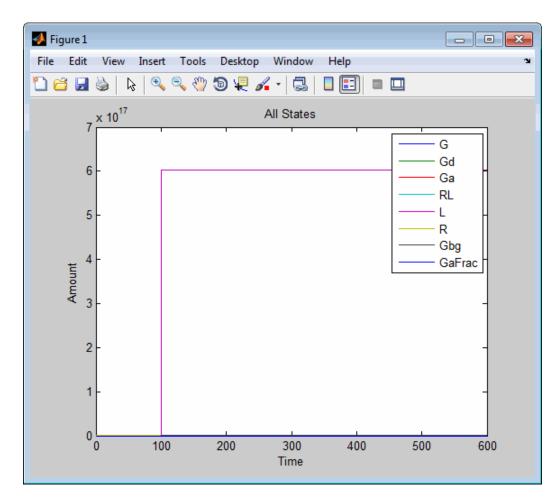
**2** Simulate the model:

```
[t,x,names] = sbiosimulate(m1);
```

**3** Plot the results:

```
plot(t,x)
legend(names)
xlabel('Time'); ylabel('Amount');title('All States')
```

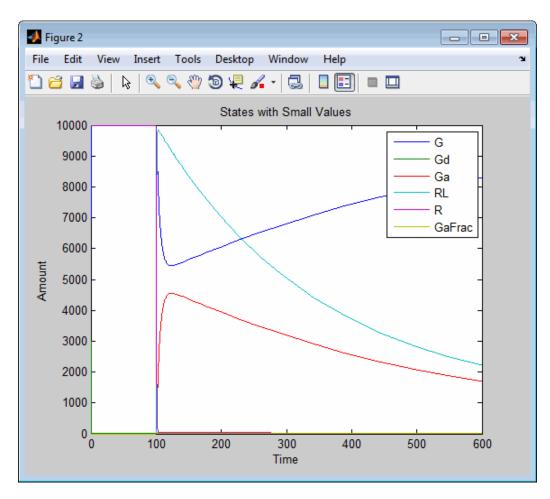
1



Notice that the species L amount increases when the event triggers at simulation time 100. The plot does not show the other species in the model due to the wide range in species amounts.

**4** To see plots of the species with smaller amounts, plot all species except the 5th (species L) and 7th (species Gbg).

```
figure; plot(t, x(:, [1:4 6 8]))
legend(names{[1:4 6 8]})
xlabel('Time'); ylabel('Amount');title('States with Small Values')
```



Notice the increase in the activation of G protein (species Ga, shown in red) after ligand (species L) is added at simulation time 100.

# Create and Simulate a Model That Changes a Parameter Value

#### In this section...

"Overview" on page  $1\mathchar`-52$ 

"Opening the Example Model" on page 1-53

"Applying an Alternate Parameter Value Using a Variant" on page 1-53

"Simulating the Model With and Without the Variant" on page 1-54

#### **Overview**

#### About the Example Model

This example uses the model described in Model of the Yeast Heterotrimeric G Protein Cycle.

This table shows the reactions used to model the G protein cycle and the corresponding rate parameters (rate constants) for each reaction. For reversible reactions, the forward rate parameter is listed first.

No.	Name	Reaction <sup>1</sup>	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1

No.	Name	Reaction <sup>1</sup>	Rate Parameters
6	G protein inactivation	Ga -> Gd	kGd

<sup>1</sup> Legend of species: L = ligand (alpha factor), R = alpha-factor receptor, Gd = inactive G-alpha-GDP, Gbg = free levels of G-beta:G-gamma complex, G = inactive Gbg:Gd complex, Ga = active G-alpha-GTP

#### About the Example

This example shows how to apply a variant to a model that contains a parameter value for a G protein cycle in a wild-type strain. The variant represents a parameter value for a G protein cycle in a mutant strain. Thus, when you simulate the model without applying the variant, you see results for the wild type strain, and when you simulate the model with the variant, you see results for the mutant strain.

#### About the Variant Created in This Example

The value of the parameter kGd is 0.11 for the wild-type strain and 0.004 for the mutant strain. To represent the mutant strain, store an alternate value of 0.004 for the kGd parameter in a variant. Then apply this variant when simulating the model.

For information on variants, see "Variant Object" on page 1-43.

## **Opening the Example Model**

Load the gprotein example project, which includes the variable m1, a model object:

```
sbioloadproject gprotein
```

The m1 model object appears in the MATLAB Workspace.

## Applying an Alternate Parameter Value Using a Variant

1 Add a variant to the m1 model object:

1

```
v1 = addvariant(m1, 'mutant_strain');
```

**2** Add content to the variant object v1. Specifically, add a parameter kGd with a value of 0.004:

```
addcontent(v1, {'parameter', 'kGd', 'Value', 0.004});
```

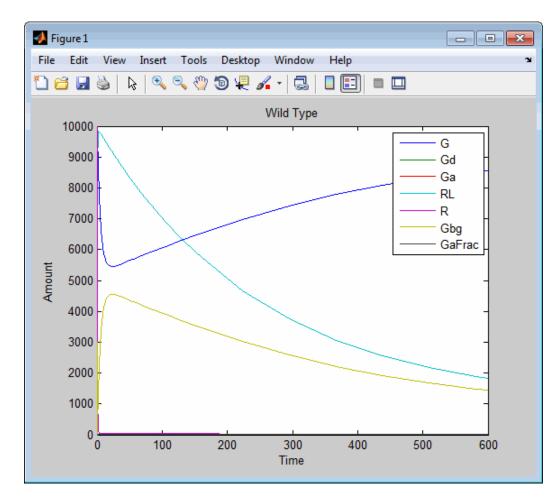
#### Simulating the Model With and Without the Variant

**1** Simulate the model of the wild-type strain, that is by not applying the variant:

```
[t,x,names] = sbiosimulate(m1);
```

**2** Plot the results of the wild-type strain (no variant):

```
plot(t,x)
legend(names)
xlabel('Time'); ylabel('Amount'); title('Wild Type')
```

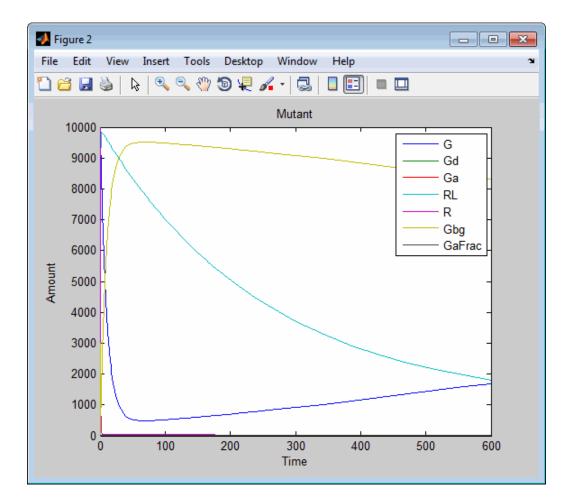


**3** Simulate the model of the mutant strain by applying the variant:

```
[t,x,names] = sbiosimulate(m1, v1);
```

**4** Plot the results of the mutant strain (variant):

```
figure; plot(t,x)
legend(names)
xlabel('Time'); ylabel('Amount'); title('Mutant')
```



## **Create and Simulate a Model with a User-Defined Function**

#### In this section...

"Overview" on page 1-57

"Creating a Custom Function" on page 1-59

"Opening the Example Model" on page 1-60

"Adding the Custom Function to the Example Model" on page 1-60

"Defining a Rule to Affect Parameter Value" on page 1-61

"Adding an Event to Reset the Solver at a Discontinuity" on page 1-61

"Simulating the Modified Model" on page 1-61

"See Also" on page 1-65

## Overview

#### **Prerequisites for the Example**

This example assumes you have a working knowledge of:

- MATLAB desktop
- Creating and saving MATLAB programs

#### About the Example Model

This example uses the model described in Model of the Yeast Heterotrimeric G Protein Cycle.

This table shows the reactions used to model the G protein cycle and the corresponding rate parameters (rate constants) for each reaction. For reversible reactions, the forward rate parameter is listed first.

No.	Name	Reaction <sup>1</sup>	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1
6	G protein inactivation	Ga -> Gd	kGd

<sup>1</sup> Legend of species: L = ligand (alpha factor), R = alpha-factor receptor, Gd = inactive G-alpha-GDP, Gbg = free levels of G-beta:G-gamma complex, G = inactive Gbg:Gd complex, Ga = active G-alpha-GTP

#### Assumptions of the Example

This example assumes that:

- An inhibitor (Inhib species) slows the inactivation of the active G protein (reaction 6 above, Ga > Gd).
- The variation in the amount of inhibitor (Inhib species) is defined in a custom function, inhibvalex.
- The inhibitor (Inhib species) affects the reaction by changing the amount of rate parameter kGd.

#### About the Example

This example shows how to create and call a custom function in a SimBiology expression. Specifically, it shows how to use a custom function in a rule expression.

#### **About Using Custom Functions in SimBiology Expressions**

You can use custom functions in:

- Reaction rate expressions (ReactionRate property)
- Rule expressions (Rule property)
- Event expressions (EventFcns property or Trigger property)

The requirements for using custom functions in SimBiology expressions are:

- Create a custom function. For more information, see function.
- Change the current folder to the folder containing your custom MATLAB file. Do this by using the cd command or by using the Current Folder field in the MATLAB desktop toolbar. Alternatively, add the folder containing your file to the search path. Do this by using the addpath command or see "Change Folders on the Search Path".
- Call the custom function in a SimBiology reaction, rule, or event expression.

**Tip** If your rule or reaction rate expression is not continuous and differentiable, see "Using Events to Address Discontinuities in Rule and Reaction Rate Expressions" on page 1-40 before simulating your model.

#### **Creating a Custom Function**

The following procedure creates a custom function, inhibvalex, which lets you specify how the inhibitor amount changes over time. The inputs are time, the initial amount of inhibitor, and a parameter that governs the amount of inhibitor. The output of the function is the amount of inhibitor.

- 1 In the MATLAB desktop, select File > New > Script, to open the MATLAB Editor.
- **2** Copy and paste the following function declaration:

```
% inhibvalex.m
function Cp = inhibvalex(t, Cpo, kel)
% This function takes the input arguments t, Cpo, and kel
% and returns the value of the inhibitor Cp.
% You can later specify the input arguments in a
% SimBiology rule expression.
```

```
% For example in the rule expression, specify:
% t as time (a keyword recognized as simulation time),
% Cpo as a parameter that represents the initial amount of inhibitor,
% and kel as a parameter that governs the amount of inhibitor.
if t < 400
    Cp = Cpo*exp(-kel*(t));
else
    Cp = Cpo*exp(-kel*(t-400));
end
```

- **3** Save the file (name the file inhibvalex.m) in a directory that is on the MATLAB search path, or to a directory that you can access.
- **4** If the location of the file is not on the MATLAB search path, change the working directory to the file location.

#### **Opening the Example Model**

Load the gprotein example project, which includes the variable m1, a model object:

```
sbioloadproject gprotein
```

The m1 model object appears in the MATLAB Workspace.

#### Adding the Custom Function to the Example Model

The following procedure creates a rule expression that calls the custom function, inhibvalex, and specifies the three input values to this function.

1 Add a repeated assignment rule to the model that specifies the three input values to the custom function, inhibvalex:

```
rule1 = addrule(m1, 'Inhib = inhibvalex(time, Cpo, Kel)',...
'repeatedAssignment');
```

The time input is a SimBiology keyword recognized as simulation time

**2** Create the two parameters used by the rule1 rule and assign values to them:

```
p1 = addparameter(m1, 'Cpo', 250);
p2 = addparameter(m1, 'Kel', 0.01);
```

**3** Create the species used by the rule1 rule:

```
s1 = addspecies(m1.Compartments, 'Inhib');
```

#### **Defining a Rule to Affect Parameter Value**

The value of rate parameter kGd is affected by the amount of inhibitor present in the system. Add a rule to the model to describe this action, but first change the ConstantValue property of the parameter kGd so that it can be varied by a rule.

1 Change the ConstantValue property of the kGd parameter to false.

```
p3 = sbioselect(m1, 'Type', 'parameter', 'Name', 'kGd');
set(p3, 'ConstantValue', false)
```

**2** Add a repeated assignment rule to the model to define how the kGd parameter is affected by the Inhib species.

```
rule2 = addrule(m1, 'kGd = 1/Inhib', 'repeatedAssignment');
```

#### Adding an Event to Reset the Solver at a Discontinuity

The custom function, inhibvalex, introduces a discontinuity in the model when time = 400. To ensure accurate simulation results, add an event to the model to reset the solver at the time of the discontinuity. Set the event to trigger at the time of the discontinuity (time = 400). The event does not need to modify the model, so create an event function that multiplies a species value by 1:

```
addevent(m1, 'time>=400', 'G=1*G');
```

#### Simulating the Modified Model

1 Configure the simulation settings (configset object) for the m1 model object to log all states during the simulation:

```
cs = getconfigset(m1);
set(cs.RuntimeOptions, 'StatesToLog', 'all')
```

1

**2** Simulate the model:

```
simDataObj = sbiosimulate(m1);
```

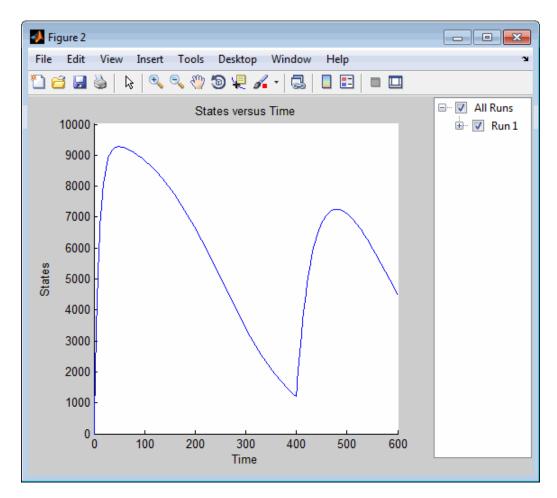
**3** Plot the results:

sbioplot(simDataObj)

The plot does not show the species of interest due to the wide range in species amounts/concentrations.

4 Plot only the species of interest. Ga:

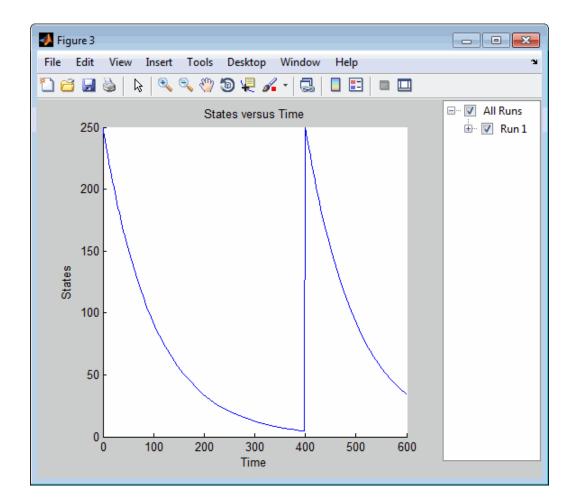
```
GaSimDataObj = selectbyname(simDataObj, 'Ga');
sbioplot(GaSimDataObj)
```



Notice the change in the profile of species Ga at time = 400 seconds (simulation time). This is the time when the inhibitor amount is changed to reflect the re-addition of inhibitor to the model.

**5** Plot only the inhibitor (Inhib species):

```
InhibSimDataObj = selectbyname(simDataObj, 'Inhib');
sbioplot(InhibSimDataObj)
```



To learn about	Refer to
Functions	function in the <i>MATLAB Function Reference</i> .
Changing the working directory to the directory containing the function file	cd in the <i>MATLAB Function Reference</i> .
Adding the directory containing function files to the MATLAB search path	addpath in the MATLAB Function Reference or "Change Folders on the Search Path" in the MATLAB Desktop Tools and Development Environment.

# See Also

1

# **View Model Equations**

You can view the system of equations that SimBiology creates when you build a model using reactions, rules, events, variants, and doses. Viewing model equations is useful for:

- Publishing purposes
- Model debugging

For details, see the getequations method of a Model object.

# Structural Analysis

- "Overview of Structural Analysis" on page 2-2
- "Model Verification" on page 2-3
- "Verifying a Model" on page 2-5
- "Conserved Moiety Determination" on page 2-6
- "Determining Conserved Moieties" on page 2-9
- "Determining the Adjacency Matrix for a Model" on page 2-13
- "Determining the Stoichiometry Matrix for a Model" on page 2-16

# **Overview of Structural Analysis**

Structural analyses let you verify and investigate the structure of your model and its quantities and expressions before actually simulating the model. These static inspections help you to:

- Confirm the model is ready for simulation.
- Better understand the relationships between quantities and expressions in the model.

For more information, see:

- "Model Verification" on page 2-3
- "Conserved Moiety Determination" on page 2-6
- "Determining the Adjacency Matrix for a Model" on page 2-13
- "Determining the Stoichiometry Matrix for a Model" on page 2-16

# **Model Verification**

#### In this section ...

"What is Model Verification?" on page 2-3

"When to Verify a Model" on page 2-3

"Verifying That a Model Has No Warnings or Errors" on page 2-4

"More About" on page 2-4

# What is Model Verification?

SimBiology has functionality that helps you find and fix warnings that you might need to be aware of, and errors that would prevent you from simulating and analyzing your model.

Model verification checks many aspects of the model including:

- Model structure
- Validity of mathematical expressions
- Dimensional analysis
- Unit conversion issues

### When to Verify a Model

You can check your model for warnings and errors at any time when constructing or working with your model. For example:

- Verify your model during construction to ensure that the model is complete.
- Verify the model after changing simulation settings, dimensional analysis settings, or unit conversion settings.

Analyses such as simulation, scanning, and parameter fitting automatically verify a model.

**Tip** Repeatedly running a task using a different variant or setting a different value for the InitialAmount property of a species, the Capacity property of a compartment, or the Value property of a parameter, generates warnings only the first time you simulate a model. Use the verification functionality described in this section to display warnings again.

# Verifying That a Model Has No Warnings or Errors

Use the verify method to see a list of warnings and errors in your model.

Use the sbiolastwarning and sbiolasterror functions to return the last warning and last error encountered during verification.

#### **More About**

For an example of verifying a model, see "Verifying a Model" on page 2-5.

# Verifying a Model

1 Create a model with a reaction that references K1, an undefined parameter:

```
% Create a model named example
model = sbiomodel('example');
% Add a compartment named cell to model
compartment = addcompartment(model, 'cell');
% Add two species, A and B, to the cell compartment
species_1 = addspecies(compartment, 'A');
species_2 = addspecies(compartment, 'B');
% Add the reaction A -> B to the model
reaction = addreaction(model, 'A -> B', 'ReactionRate', 'K1');
```

**2** Verify the model to check for warnings and errors:

verify(model)

```
??? --> Error reported from Expression Validation:
The name 'K1' in reaction 'A -> B' does not refer to any in-scope species,
parameters, or compartments.
```

**3** Address the error by defining the parameter K1:

```
% Add a parameter, K1, to the model with a value of 3
parameter = addparameter(model, 'K1', 3);
```

**4** Verify the model again:

verify(model)

# **Conserved Moiety Determination**

#### In this section ...

"Introduction to Moiety Conservation" on page 2-6

"Algorithms for Conserved Cycle Calculations" on page 2-6

"More About" on page 2-8

### Introduction to Moiety Conservation

*Conserved moieties* represent quantities that are conserved in a system, regardless of the individual reaction rates.

Consider this simple network:

reaction 1: A -> B reaction 2: B -> C reaction 3: C -> A

Regardless of the rates of reactions 1, 2, and 3, the quantity A + B + C is conserved throughout the dynamic evolution of the system. This conservation is termed structural because it depends only on the structure of the network, rather than on details such as the kinetics of the reactions involved. In the context of systems biology, such a conserved quantity is sometimes referred to as a conserved moiety. A typical, real-world example of a conserved moiety is adenine in its various forms ATP, ADP, AMP, etc. Finding and analyzing conserved moieties can yield insights into the structure and function of a biological network. In addition, for the quantitative modeler, conserved moieties represent dependencies that can be removed to reduce a system's dimensionality, or number of dynamic variables. In the previous simple network, in principle, it is only necessary to calculate the time courses for A and B. After this is done, C is fixed by the conservation relation.

# **Algorithms for Conserved Cycle Calculations**

The sbioconsmolety function analyzes conservation relationships in a model by calculating a complete set of linear conservation relations for the species in the model object.

2-6

**sbioconsmoiety** lets you specify one of three algorithms based on the nature of the model and the required results:

- 'qr' sbioconsmoiety uses an algorithm based on QR factorization. From a numerical standpoint, this is the most efficient and reliable approach.
- 'rreduce' sbioconsmoiety uses an algorithm based on row reduction, which yields better numbers for smaller models. This is the default.
- 'semipos' sbioconsmolety returns conservation relations in which all the coefficients are greater than or equal to zero, permitting a more transparent interpretation in terms of physical quantities.

For larger models, the QR-based method is recommended. For smaller models, row reduction or the semipositive algorithm may be preferable. For row reduction and QR factorization, the number of conservation relations returned equals the row rank degeneracy of the model object's stoichiometry matrix. The semipositive algorithm can return a different number of relations. Mathematically speaking, this algorithm returns a generating set of vectors for the space of semipositive conservation relations.

In some situations, you may be interested in the dimensional reduction of your model via conservation relations. Recall the simple model, presented in "Introduction to Moiety Conservation" on page 2-6, that contained the conserved cycle A + B + C. Given A and B, C is determined by the conservation relation; the system can be thought of as having only two dynamic variables rather than three. The 'link' algorithm specification caters to this situation. In this case, sbioconsmoiety partitions the species in the model into independent and dependent sets and calculates the dependence of the dependent species on the independent species.

Consider a general system with an n-by-m stoichiometry matrix N of rank k, and suppose that the rows of N are permuted (which is equivalent to permuting the species ordering) so that the first k rows are linearly independent. The last n - k rows are then necessarily dependent on the first k rows.

The matrix N can be split into the following independent and dependent parts,

$$\mathbf{N} = \begin{pmatrix} N_R \\ N_D \end{pmatrix}$$

where R in the independent submatrix  $N_R$  denotes 'reduced'; the (n-k)-by-k link matrix L0 is defined so that  $N_D$  =  $L0^*N_R$ . In other words, the link matrix gives the dependent rows  $N_D$  of the stoichiometry matrix, in terms of the independent rows  $N_R$ . Because each row in the stoichiometry matrix corresponds to a species in the model, each row of the link matrix encodes how one dependent species is determined by the k independent species.

# **More About**

For examples of determining conserved moieties, see:

- "Determining Conserved Moieties" on page 2-9
- Finding Conserved Quantities in a Pathway Model

# **Determining Conserved Moieties**

1 Load the Goldbeter Mitotic Oscillator project, which includes the variable m1, a model object:

sbioloadproject Goldbeter\_Mitotic\_Oscillator\_with\_reactions

The m1 model object appears in the MATLAB Workspace.

**2** Display the species information:

m1.Compartments.Species

SimBiology Species Array

Index:	Compartment: unnamed	Name: C	InitialAmount: 0.01	InitialAmountUnits:
2	unnamed	M	0.01	
3	unnamed	Mplus	0.99	
4	unnamed	Mt	1	
5	unnamed	Х	0.01	
6	unnamed	Xplus	0.99	
7	unnamed	Xt	1	
8	unnamed	V1	0	
9	unnamed	V3	0	
10	unnamed	AA	0	

**3** Display the reaction information:

m1.Reactions

SimBiology Reaction Array

Index: 1	Reaction: AA -> C
•	• • •
2	C -> AA
3	C + X -> AA + X
4	Mplus + C $\rightarrow$ M + C
5	M -> Mplus
6	Xplus + M -> X + M
7	X -> Xplus

**4** Use the simplest form of the sbioconsmoiety function and display the results. The default call to sbioconsmoiety, in which no algorithm is specified, uses an algorithm based on row reduction.

0

0

1

```
[g sp] = sbioconsmoiety(m1)
g =
      0
             1
                            0
                                   0
                    1
      0
             0
                    0
                            1
                                   1
      0
             0
                    0
                            0
                                   0
sp =
     'C'
     'M'
     'Mplus'
     'X'
     'Xplus'
     ' AA '
```

The columns in g are labeled by the species sp. Thus the first row describes the conserved relationship, M + Mplus. Notice that the third row indicates that the species AA is conserved, which is because AA is constant (ConstantAmount = 1).

**5** Call sbioconsmolety again, this time specifying the semipositive algorithm to explore conservation relations in the model. Also specify to return the conserved moleties in a cell array of strings, instead of a matrix.

```
cons_rel = sbioconsmoiety(m1,'semipos','p')
cons_rel =
    'AA'
    'X + Xplus'
    'M + Mplus'
```

6 Use the 'link' option to study the dependent and independent species.

[SI,SD,LO,NR,ND] = sbioconsmoiety(m1, 'link');

**7** Show the list of independent species:

SI = 'C' 'M' 'X'

8 Show the list of dependent species:

SD

SD = 'Mplus' 'Xplus' 'AA'

**9** Show the link matrix relating SD and SI by converting the LO output from a sparse matrix to a full matrix:

LO\_full = full(LO) LO\_full =

0	-1.0000	0
0	0	-1.0000
0	0	0

10 Show the independent stoichiometry matrix,  $N_R$  by converting the NR output from a sparse matrix to a full matrix:

NR\_full = full(NR) NR\_full = 1 -1 -1 0 0 0 0 0 0 0 1 -1 0 0

- 0 0 0 0 0 1 -1
- 11 Show the dependent stoichiometry matrix,  $N_{\rm D}$  by converting the ND output from a sparse matrix to a full matrix:

ND\_full = full(ND)

ND\_full =

0	0	0	- 1	1	0	0
0	0	0	0	0	- 1	1
0	0	0	0	0	0	0

# Determining the Adjacency Matrix for a Model

#### In this section...

"What Is an Adjacency Matrix?" on page 2-13

"Retrieving an Adjacency Matrix for a Model" on page 2-14

#### What Is an Adjacency Matrix?

An *adjacency matrix* lets you easily determine:

- The reactants and products in a specific reaction in a model
- The reactions that a specific species is part of, and whether the species is a reactant or product in that reaction

An adjacency matrix is an *N*-by-*N* matrix, where *N* equals the total number of species and reactions in a model. Each row corresponds to a species or reaction, and each column corresponds to a species or reaction.

The matrix indicates which species and reactions are involved as reactants and products:

- Reactants are represented in the matrix with a 1 at the appropriate location (row of species, column of reaction). Reactants appear above the diagonal.
- Products are represented in the matrix with a 1 at the appropriate location (row of reaction, column of species). Products appear below the diagonal.
- All other locations in the matrix contain a 0.

For example, if a model object contains one reaction equal to  $A + B \rightarrow C$  and the Name property of the reaction is R1, the adjacency matrix is:

	А	В	С	R1
А	0	0	0	1
В	0	0	0	1
С	0	0	0	0
R1	0	0	1	0

# **Retrieving an Adjacency Matrix for a Model**

Retrieve an adjacency matrix for a model by passing the model object as an input argument to the getadjacencymatrix method.

1 Read in m1, a model object, using sbmlimport:

```
m1 = sbmlimport('lotka.xml');
```

**2** Get the adjacency matrix for m1:

[M, Headings] = getadjacencymatrix(m1)

М =

(5,1)	1
(5,2)	1
(6,3)	1
(7,4)	1
(1,5)	1
(2,5)	1
(2,6)	1
(3,6)	1
(3,7)	1

Headings = 'x' 'v1' 'v2' '7' 'Reaction1' 'Reaction2' 'Reaction3'

**3** Convert the adjacency matrix from a sparse matrix to a full matrix to more easily see the relationships between species and reactions:

M full = full(M)

M\_full =

0	0	0	0	1	0	0
0	0	0	0	1	1	0
0	0	0	0	0	1	1
0	0	0	0	0	0	0
1	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0

# **Determining the Stoichiometry Matrix for a Model**

#### In this section...

"What Is a Stoichiometry Matrix?" on page 2-16

"Retrieving a Stoichiometry Matrix for a Model" on page 2-17

#### What Is a Stoichiometry Matrix?

A stoichiometry matrix lets you easily determine:

- The reactants and products in a specific reaction in a model, including the stoichiometric value of the reactants and products
- The reactions that a specific species is part of, and whether the species is a reactant or product in that reaction

A stoichiometry matrix is an M-by-N matrix, where M equals the total number of reactions in a model, and N equals the total number of species in a model. Each row corresponds to a species, and each column corresponds to a reaction.

The matrix indicates which species and reactions are involved as reactants and products:

- Reactants are represented in the matrix with their stoichiometric value at the appropriate location (row of species, column of reaction). Reactants appear as negative values.
- Products are represented in the matrix with their stoichiometric value at the appropriate location (row of species, column of reaction). Products appear as positive values.
- All other locations in the matrix contain a **0**.

For example, consider a model object containing two reactions. One reaction (named R1) is equal to  $2 A + B \rightarrow 3 C$ , and the other reaction (named R2) is equal to  $B + 3 D \rightarrow 4 A$ . The stoichiometry matrix is:

	R1	R2
A	- 2	4
В	- 1	- 1

C 3 0 D 0 -3

### **Retrieving a Stoichiometry Matrix for a Model**

Retrieve a stoichiometry matrix for a model by passing the model object as an input argument to the getstoichmatrix method.

1 Read in m1, a model object, using sbmlimport:

m1 = sbmlimport('lotka.xml');

2 Get the stoichiometry matrix for m1:

[M,objSpecies,objReactions] = getstoichmatrix(m1)

M =

(2,1)	1
(2,2)	- 1
(3,2)	1
(3,3)	- 1
(4,3)	1

objSpecies =

'x' 'y1' 'y2' 'z'

objReactions = 'Reaction1' 'Reaction2'

- 'Reaction3'
- **3** Convert the stoichiometry matrix from a sparse matrix to a full matrix to more easily see the relationships between species and reactions:

M\_full = full(M)

M\_full =

0	0	0
1	- 1	0
0	1	- 1
0	0	1

# Simulation and Analysis

- "Overview of Simulation and Analysis" on page 3-2
- "Model Simulation" on page 3-3
- "Simulate a Model and View Results" on page 3-12
- "Sensitivity Calculation" on page 3-18
- "Calculate Sensitivities" on page 3-22
- "Perform Parameter Scan" on page 3-27
- "Parameter Estimation" on page 3-29
- "Estimate Parameters" on page 3-32
- "Accelerating Model Simulations and Analyses" on page 3-44

# **Overview of Simulation and Analysis**

After creating models in SimBiology, you can simulate and analyze them.

# **Typical Workflow**

To simulate a model, SimBiology:

- **1** Converts the model expressions and quantities to a system of differential equations.
- **2** Uses deterministic or stochastic solvers to numerically solve these equations.
- **3** Determines the changes in species amounts and parameter values over time.

For more information, see "Model Simulation" on page 3-3.

SimBiology also lets you analyze models. These analyses include a basic simulation of the model as well as additional evaluations such as:

- "Sensitivity Calculation" on page 3-18
- "Perform Parameter Scan" on page 3-27
- "Parameter Estimation" on page 3-29

# **Model Simulation**

In this	s section
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"Simulating a Model" on page 3-3
"Plotting Simulation Results" on page 3-4
"Interpreting Simulation Results" on page 3-4
"Configuring Stop Time and Other Simulation Settings" on page 3-4
"Choosing a Simulation Solver" on page 3-5
"SUNDIALS Solvers" on page 3-6
"Stochastic Solvers" on page 3-6
"Ensemble Runs of Stochastic Simulations" on page 3-11
"See Also" on page 3-11

# Simulating a Model

Simulate a model by providing the model object as an input argument to the sbiosimulate function.

When you simulate a model, you can return results (time points, state data, and state names) in two forms:

- Three separate arrays
- One SimData object

A SimData object also includes metadata such as the types and names for the logged states, the configuration set used during simulation, and the date of the simulation. It is a convenient way of keeping time data, state data, and associated metadata together. A SimData object has associated properties and methods, which you can use to access and manipulate the data.

For more information on simulating a model, see "Simulate a Model and View Results" on page 3-12.

### **Plotting Simulation Results**

If you return time and state data from a simulation in three output arguments, you can use these arguments as inputs to the plot function to view your results. For more information, see sbiosimulate.

If you return time and state data from a simulation in a SimData object, you can use the SimData object as an input to the sbioplot function to view your results.

For more information on plotting simulation results, see "Simulate a Model and View Results" on page 3-12.

### **Interpreting Simulation Results**

After running a simulation, you may see negative amounts or concentrations for species in the results plot or data array. These negative values can be either:

- Slightly negative due to numerical noise introduced by the simulation process. In this case, you can interpret these values as 0.
- Significantly negative due to the dynamics in your model not being physical, that is, the dynamics in the system are driving a particular species to be negative. In this case, examine your reaction rate expressions to ensure they implement correct dynamics.

### **Configuring Stop Time and Other Simulation Settings**

A model has a configuration set (Configset object) associated with it to control the simulation. You can edit the properties of a Configset object to control all aspects of the simulation, including:

- Stop time (StopTime, MaximumNumberOfLogs, and MaximumWallClock properties)
- Time units (TimeUnits property)
- Solver and error tolerances (SolverType and SolverOptions properties)
- Maximum time step size (MaxStep property)
- Data to record (RuntimeOptions property)

- Frequency of data recording (OutputTimes and LogDecimation properties)
- Sensitivity analysis (SensitivityAnalysisOptions and SolverOptions properties)
- Dimensional analysis and unit conversion (CompileOptions property)

To view the Configset object, provide the model object as an input argument to the getconfigset method.

To edit the properties of a Configset object, use the set method.

For more information on viewing and editing the stop time and other simulation settings, see "Simulate a Model and View Results" on page 3-12.

#### **Choosing a Simulation Solver**

To simulate a model, the SimBiology software converts a model to a system of differential equations. It then uses a solver function to compute solutions for these equations at different time intervals, giving the model's states and outputs over a span of time.

Available solvers are:

- **ODE Solvers** These include Nonstiff Deterministic Solvers and Stiff Deterministic Solvers. The solver functions implement numerical integration methods for solving initial value problems for ordinary differential equations (ODEs). Beginning at the initial time with initial conditions, they step through the time interval, computing a solution at each time step. If the solution for a time step satisfies the solver's error tolerance criteria, it is a successful step. Otherwise, it is a failed attempt; the solver shrinks the step size and tries again. For more information, see "ODE Solvers" in the MATLAB Mathematics documentation.
- SUNDIALS Solvers At a fundamental level the core algorithms for the SUNDIALS solvers are similar to those for some of the solvers in the MATLAB ODE suite and work as described above in ODE Solvers. For more information, see "SUNDIALS Solvers" on page 3-6.
- Stochastic Solvers Use with models containing a small number of molecules. Stochastic solvers include stochastic simulation algorithm,

explicit tau-leaping algorithm, and implicit tau-leaping algorithm. For more information, see "Stochastic Solvers" on page 3-6.

### **SUNDIALS Solvers**

SUNDIALS (Suite of Nonlinear and Differential/Algebraic Equation Solvers) are part of a freely available third-party package developed at Lawrence Livermore National Laboratory. All other ODE solvers used for simulation of SimBiology models, such as ode45 and ode15s, are part of the MATLAB ODE suite.

When you specify sundials for the solver, the software chooses one of two SUNDIALS solvers, CVODE or IDA, as appropriate for your model:

- **CVODE** is a solver for systems of ODEs, both nonstiff and stiff. This is used when a model has no algebraic rules.
- **IDA** is a differential-algebraic equation (DAE) solver, used when one or more algebraic rules are present.

For more information on the SUNDIALS solvers, see http://www.llnl.gov/casc/sundials/description/description.html.

#### **Stochastic Solvers**

- "When to Use Stochastic Solvers" on page 3-7
- "Model Prerequisites for Simulating with a Stochastic Solver" on page 3-7
- "What Happens During a Stochastic Simulation?" on page 3-7
- "Stochastic Simulation Algorithm (SSA)" on page 3-8
- "Explicit Tau-Leaping Algorithm" on page 3-8
- "Implicit Tau-Leaping Algorithm" on page 3-9
- "More About" on page 3-10
- "References" on page 3-10

#### When to Use Stochastic Solvers

The stochastic simulation algorithms provide a practical method for simulating reactions that are stochastic in nature. Models with a small number of molecules can realistically be simulated stochastically, that is, allowing the results to contain an element of probability, unlike a deterministic solution.

#### Model Prerequisites for Simulating with a Stochastic Solver

Model prerequisites include:

- All reactions in the model must have their KineticLaw property set to MassAction.
- If your model contains events, you can simulate using the stochastic ssa solver. Other stochastic solvers do not support events.
- Your model must not contain doses. No stochastic solvers support doses.

Additionally, if you perform an individual or population fitting on a model whose configset object specifies a stochastic solver and options, be aware that during the fitting SimBiology temporarily changes:

- SolverType property to the default solver of ode15s
- SolverOptions property to the options last configured for a deterministic solver

#### What Happens During a Stochastic Simulation?

During a stochastic simulation of a model, the software ignores any rate, assignment, or algebraic rules if present in the model. Depending on the model, stochastic simulations can require more computation time than deterministic simulations.

**Tip** When simulating a model using a stochastic solver, you can increase the LogDecimation property of the configset object to record fewer data points and decrease run time.

#### Stochastic Simulation Algorithm (SSA)

The Chemical Master Equation (CME) describes the dynamics of a chemical system in terms of the time evolution of probability distributions. Directly solving for this distribution is impractical for most realistic problems. The stochastic simulation algorithm (SSA) instead efficiently generates individual simulations that are consistent with the CME, by simulating each reaction using its propensity function. Thus, analyzing multiple stochastic simulations to determine the probability distribution is more efficient than directly solving the CME.

#### Advantage

• This algorithm is exact.

#### Disadvantages

- Because this algorithm evaluates one reaction at a time, it might be too slow for models with a large number of reactions.
- If the number of molecules of any reactants is huge, it might take a long time to complete the simulation.

#### **Explicit Tau-Leaping Algorithm**

Because the stochastic simulation algorithm might be too slow for many practical problems, this algorithm was designed to speed up the simulation at the cost of some accuracy. The algorithm treats each reaction as being independent of the others. It automatically chooses a time interval such that the relative change in the propensity function for each reaction is less than your error tolerance. After selecting the time interval, the algorithm computes the number of times each reaction occurs during the time interval and makes the appropriate changes to the concentration of various chemical species involved.

#### Advantages

- This algorithm can be orders of magnitude faster than the SSA.
- You can use this algorithm for large problems (if the problem is not numerically stiff).

#### Disadvantages

- This algorithm sacrifices some accuracy for speed.
- This algorithm is not good for stiff models.
- You need to specify the error tolerance so that the resulting time steps are of the order of the fastest time scale.

#### Implicit Tau-Leaping Algorithm

Like the explicit tau-leaping algorithm, the implicit tau-leaping algorithm is also an approximate method of simulation designed to speed up the simulation at the cost of some accuracy. It can handle numerically stiff problems better than the explicit tau-leaping algorithm. For deterministic systems, a problem is said to be numerically stiff if there are "fast" and "slow" time scales present in the system. For such problems, the explicit tau-leaping method performs well only if it continues to take small time steps that are of the order of the fastest time scale. The implicit tau-leaping method can potentially take much larger steps and still be stable. The algorithm treats each reaction as being independent of others. It automatically selects a time interval such that the relative change in the propensity function for each reaction is less than the user-specified error tolerance. After selecting a time interval, the algorithm computes the number of times each reaction occurs during the time interval and makes the appropriate changes to the concentration of various chemical species involved.

#### Advantages

- This algorithm can be much faster than the SSA. It is also usually faster than the explicit tau-leaping algorithm.
- You can use this algorithm for large problems and also for numerically stiff problems.
- The total number of steps taken is usually less than the explicit-tau-leaping algorithm.

#### Disadvantages

• This algorithm sacrifices some accuracy for speed.

- There is a higher computational burden for each step as compared to the explicit tau-leaping algorithm. This leads to a larger CPU time per step.
- This method often dampens perturbations of the slow manifold leading to a reduced state variance about the mean.

#### **More About**

- "Ensemble Runs of Stochastic Simulations" on page 3-11
- Analysis of Stochastic Ensemble Data in SimBiology example

#### References

[1] Gibson M.A., Bruck J. (2000), "Exact Stochastic Simulation of Chemical Systems with Many Species and Many Channels," Journal of Physical Chemistry, 105:1876–1899.

[2] Gillespie D. (1977), "Exact Stochastic Simulation of Coupled Chemical Reactions," The Journal of Physical Chemistry, 81(25): 2340–2361.

[3] Gillespie D. (2000), "The Chemical Langevin Equation," Journal of Chemical Physics, 113(1): 297–306.

[4] Gillespie D. (2001), "Approximate Accelerated Stochastic Simulation of Chemically Reacting Systems," Journal of Chemical Physics,115(4):1716–1733.

[5] Gillespie D., Petzold L. (2004), "Improved Leap-Size Selection for Accelerated Stochastic Simulation," Journal of Chemical Physics, 119:8229–8234

[6] Rathinam M., Petzold L., Cao Y., Gillespie D. (2003), "Stiffness in Stochastic Chemically Reacting Systems: The Implicit Tau-Leaping Method," Journal of Chemical Physics, 119(24):12784–12794.

[7] Moler, C. (2003), "Stiff Differential Equations Stiffness is a subtle, difficult, and important concept in the numerical solution of ordinary differential equations," MATLAB News & Notes.

## **Ensemble Runs of Stochastic Simulations**

Because stochastic simulations rely on an element of probability, sequential runs produce different results. Therefore, multiple stochastic runs are needed to determine the probability distribution of the simulation results.

Ensemble runs perform multiple simulations of a model using a stochastic solver. They let you gather data from multiple stochastic runs of the model so you can compare and analyze fluctuations in the behavior of a model over repeated stochastic simulations.

#### **Running Ensemble Simulations**

The following functions let you perform and analyze ensemble runs at the command line:

- sbioensemblerun Perform a stochastic ensemble run of the MATLAB model object.
- sbioensembleplot Show a 2-D distribution plot or a 3-D shaded plot of the time varying distribution of one or more specified species.
- sbioensemblestats Get mean and variance as a function of time for all the species in the model used to generate ensemble data by running sbioensemblerun.

## See Also

For examples of simulating models, see:

- "Simulate a Model and View Results" on page 3-12
- Analysis of Stochastic Ensemble Data in SimBiology example

# Simulate a Model and View Results

In this section.	••
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"Overview" on page 3-12

"Loading the Example Model" on page 3-13

"Configuring Simulation Settings" on page 3-13

"Simulating the Model" on page 3-14

"Plotting Simulation Results" on page 3-14

"Extracting Data for Analysis" on page 3-17

## Overview

### About the Example Model

This example uses the model described in "Model of the Yeast Heterotrimeric G Protein Cycle" on page C-19 to illustrate model simulation.

This table shows the reactions used to model the G protein cycle and the corresponding rate parameters (rate constants) for each mass action reaction. For reversible reactions, the forward rate parameter is listed first.

No.	Name	Reaction <sup>1</sup>	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1

3-12

No.	Name	Reaction <sup>1</sup>	Rate Parameters
6	G protein inactivation	Ga -> Gd	kGd

<sup>1</sup> Legend of species: L = ligand (alpha factor), R = alpha-factor receptor, Gd = inactive G-alpha-GDP, Gbg = free levels of G-beta:G-gamma complex, G = inactive Gbg:Gd complex, Ga = active G-alpha-GTP

### About the Example

This example shows how to configure simulation settings and simulate a model, saving the results in a SimData object. It then illustrates how to plot all species in the SimData object, as well as plot only the species or parameter of interest, such as:

- GaFrac The fraction of total Ga that is active
- Ga, G, and Gd The species that contain G-alpha units

The example also illustrates how to extract data from the SimData object for analysis.

## Loading the Example Model

Load the gprotein.sbproj project, which includes the variable m1, a model object:

sbioloadproject gprotein

The m1 model object appears in the MATLAB Workspace.

## **Configuring Simulation Settings**

Set the simulation solver to ode15s and set a stop time of 500 by editing the SolverType and StopTime properties of the configset object associated with the m1 model:

```
csObj = getconfigset(m1);
set(csObj, 'SolverType', 'ode15s', 'StopTime', 500)
```

## Simulating the Model

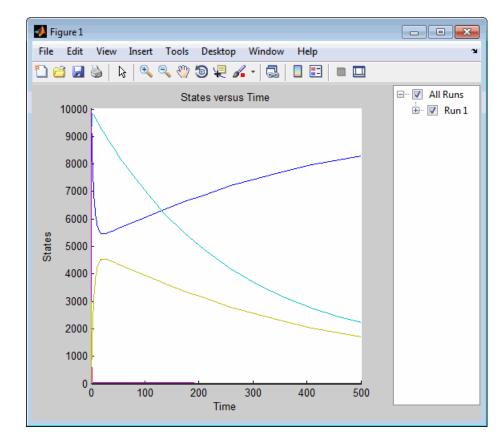
Simulate the model, saving the results in a SimData object:

```
simDataObj = sbiosimulate(m1);
```

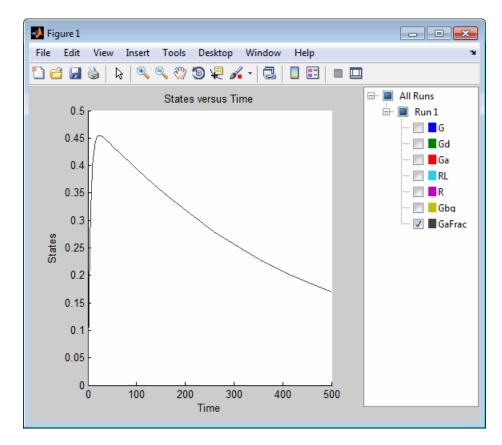
## **Plotting Simulation Results**

**1** Plot all the species in the model:

```
sbioplot(simDataObj)
```

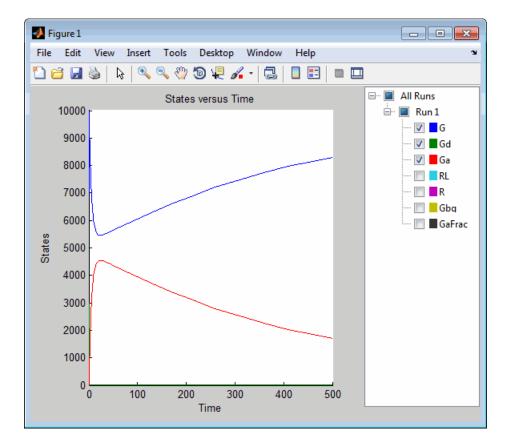


2 Show only the plot of the GaFrac parameters by expanding Run 1, and then clearing all check boxes other than GaFrac:



Notice the scale of the *y*-axis changes as you remove species from the plot.

**3** Show only the plots of the species containing G-alpha units by selecting the G, Gd, and Ga check boxes, and clearing the GaFrac check box:



**4** Alternately, create the previous plots by using the selectbyname method to create SimData objects containing data for only the species of interest, and then plotting these smaller objects:

```
GafracObj = selectbyname(simDataObj, 'GaFrac');
GaAllObj = selectbyname(simDataObj, {'Ga', 'G', 'Gd'});
sbioplot(GafracObj)
sbioplot(GaAllObj)
```

## **Extracting Data for Analysis**

1 Use the selectbyname method to extract arrays containing the time points, state data, and state names for a subset of the data, namely the Ga, G, and Gd species that contain G-alpha units:

```
[times, data, names] = selectbyname(simDataObj, {'Ga', 'G', 'Gd'});
```

**2** The previous plot shows that at the start of the simulation, the total G-alpha units (G + Gd + Ga) = 10000. Check the conservation of mass of the G-alpha units by summing their values to see if they equal 10000:

```
GaTotal = sum(data, 2);
```

**3** View the sum of the values for the G-alpha units for the last 10 time points in the simulation:

```
GaTotal(end-9:end)
ans =
1.0e+004 *
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000
1.0000
```

# **Sensitivity Calculation**

#### In this section...

"About Calculating Sensitivities" on page 3-18

"Model Requirements for Calculating Sensitivities" on page 3-18

"Setting SolverOptions Properties for Calculating Sensitivities" on page 3-20

"Calculating Sensitivities of Model and Viewing Results" on page 3-20

"See Also" on page 3-21

"References" on page 3-21

## **About Calculating Sensitivities**

Calculating sensitivities lets you determine which species or parameter in a model is most sensitive to a specific condition (for example, a drug), defined by a species or parameter. Calculating sensitivities calculates the time-dependent sensitivities of all the species states with respect to species initial conditions and parameter values in the model.

Thus, if a model has a species x, and two parameters y and z, the time-dependent sensitivities of x with respect to each parameter value are the time-dependent derivatives

 $\frac{\partial x}{\partial y}, \frac{\partial x}{\partial z}$ 

where, the numerator is the sensitivity output and the denominators are the sensitivity inputs to sensitivity analysis.

For more information on the calculations performed, see "References" on page 3-21.

## **Model Requirements for Calculating Sensitivities**

Sensitivity analysis is supported only by the ordinary differential equation (ODE) solvers. The software calculates local sensitivities by combining the

original ODE system for a model with the auxiliary differential equations for the sensitivities. The additional equations are derivatives of the original equations with respect to parameters. This method is sometimes called "forward sensitivity analysis" or "direct sensitivity analysis". This larger system of ODEs is solved simultaneously by the solver.

SimBiology sensitivity analysis uses "complex-step approximation" to calculate derivatives of reaction rates. This technique yields accurate results for the vast majority of typical reaction kinetics, which involve only simple mathematical operations and functions. When a reaction rate involves a nonanalytic function, this technique can lead to inaccurate results. In this case, either sensitivity analysis is disabled, or sensitivity analysis warns you that the computed sensitivities may be inaccurate. An example of such a nonanalytic function is the MATLAB function abs. If sensitivity analysis gives questionable results on a model whose reaction rates contain unusual functions, you may be running into limitations of the complex-step method. Contact MathWorks Technical Support for additional information.

**Note** Models containing the following active components do not support sensitivity analysis:

- Nonconstant compartments
- Algebraic rules
- Events

**Note** You can perform sensitivity analysis on a model containing repeated assignment rules, but only if the repeated assignment rules do not determine species or parameters used as inputs or outputs in sensitivity analysis.

# Setting SolverOptions Properties for Calculating Sensitivities

Perform sensitivity analysis at the command line by setting the following properties of the SolverOptions property of your configset object, before using the sbiosimulate function:

- SensitivityAnalysis Set to true to calculate the time-dependent sensitivities of all the species states defined by the Outputs property with respect to the initial conditions of the species and the values of the parameters specified in Inputs.
- SensitivityAnalysisOptions An object that holds the sensitivity analysis options in the configuration set object. Properties of SensitivityAnalysisOptions are:
  - Outputs Specify the species and parameters for which you want to compute the sensitivities. This is the numerator as described in "About Calculating Sensitivities" on page 3-18.
  - Inputs Specify the species and parameters with respect to which you want to compute the sensitivities. Sensitivities are calculated with respect to the InitialAmount property of the specified species. This is the denominator, described in "About Calculating Sensitivities" on page 3-18.
  - Normalization Specify the normalization for the calculated sensitivities:
    - 'None' No normalization
    - · 'Half' Normalization relative to the numerator (species output) only
    - · 'Full' Full dedimensionalization

For more information about normalization, see Normalization in the *SimBiology Reference*.

## Calculating Sensitivities of Model and Viewing Results

After setting SolverOptions properties, calculate the sensitivities of a model by providing the model object as an input argument to the sbiosimulate function.

The sbiosimulate function returns a SimData object containing the following simulation data:

- Time points, state data, state names, and sensitivity data
- Metadata such as the types and names for the logged states, the configuration set used during simulation, and the date of the simulation

A SimData object is a convenient way of keeping time data, state data, sensitivity data, and associated metadata together. A SimData object has properties and methods associated with it, which you can use to access and manipulate the data.

## See Also

For examples of calculating sensitivities, see:

- "Calculate Sensitivities" on page 3-22
- Parameter Scanning, Parameter Estimation, and Sensitivity Analysis in the Yeast Heterotrimeric G Protein Cycle

## References

Ingalls, B.P., and Sauro, H.M. (2003). Sensitivity analysis of stoichiometric networks: an extension of metabolic control analysis to non-steady state trajectories. J Theor Biol. *222(1)*, 23–36.

Martins, J.R.R.A., Sturdza, P., and Alanso, J.J. (Jan. 2001). The connection between the complex-step derivative approximation and algorithmic differentiation. AIAA Paper 2001–0921.

Martins, J.R.R.A., Kroo, I.M., and Alanso, J.J. (Jan. 2000). An automated method for sensitivity analysis using complex variables. AIAA Paper 2000–0689.

# **Calculate Sensitivities**

#### In this section...

"Overview" on page 3-22

"Loading and Configuring the Model for Sensitivity Analysis" on page 3-23

"Performing Sensitivity Analysis" on page 3-24

"Extracting and Plotting Sensitivity Data" on page 3-24

## Overview

#### About the Example Model

This example uses the model described in "Model of the Yeast Heterotrimeric G Protein Cycle" on page C-19 to illustrate SimBiology sensitivity analysis options.

This table lists the reactions used to model the G protein cycle and the corresponding rate parameters (rate constants) for each mass action reaction. For reversible reactions, the forward rate parameter is listed first.

No.	Name	Reaction <sup>1</sup>	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1

No.	Name	Reaction <sup>1</sup>	Rate Parameters
6	G protein inactivation	Ga -> Gd	kGd

<sup>1</sup> Legend of species: L = ligand (alpha factor), R = alpha-factor receptor, Gd = inactive G-alpha-GDP, Gbg = free levels of G-beta:G-gamma complex, G = inactive Gbg:Gd complex, Ga = active G-alpha-GTP

#### About the Example

Assume that you are calculating the sensitivity of species **Ga** with respect to every parameter in the model. Thus, you want to calculate the time-dependent derivatives

 $\frac{\partial(Ga)}{\partial(kRLm)}, \frac{\partial(Ga)}{\partial(kRL)}, \frac{\partial(Ga)}{\partial(kG1)}, \frac{\partial(Ga)}{\partial(kGa)} \cdots$ 

## Loading and Configuring the Model for Sensitivity Analysis

1 The gprotein\_norules.sbproj project contains a model that represents the wild-type strain (stored in variable m1). Load the G protein model for the wild-type strain:

sbioloadproject gprotein\_norules m1

The m1 model object appears in the MATLAB Workspace.

**2** The options for sensitivity analysis are in the configuration set object. Get the configuration set object from the model:

```
csObj = getconfigset(m1);
```

**3** Use the sbioselect function, which lets you query by type, to retrieve the Ga species from the model:

```
Ga = sbioselect(m1,'Type','species','Where','Name','==','Ga');
```

**4** Set the Outputs property of the SensitivityAnalysisOptions object to the Ga species:

```
set(csObj.SensitivityAnalysisOptions,'Outputs', Ga);
```

**5** Use the sbioselect function, which lets you query by type, to retrieve all the parameters from the model and store the vector in a variable, pif:

```
pif = sbioselect(m1, 'Type', 'parameter');
```

**6** Set the Inputs property of the SensitivityAnalysisOptions object to the pif variable containing the parameters:

set(csObj.SensitivityAnalysisOptions,'Inputs', pif);

7 Enable sensitivity analysis in the configuration set object (csObj) by setting the SensitivityAnalysis option to true:

set(csObj.SolverOptions, 'SensitivityAnalysis', true);

8 Set the Normalization property of the SensitivityAnalysisOptions object to perform 'Full' normalization:

set(csObj.SensitivityAnalysisOptions,'Normalization', 'Full');

## **Performing Sensitivity Analysis**

Simulate the model and return the data to a SimData object:

simDataObj = sbiosimulate(m1);

## **Extracting and Plotting Sensitivity Data**

You can extract sensitivity results using the getsensmatrix method of a SimData object. In this example, R is the sensitivity of the species Ga with respect to eight parameters. This example shows how to compare the variation of sensitivity of Ga with respect to various parameters, and find the parameters that affect Ga the most.

1 Extract sensitivity data in output variables T (time), R (sensitivity data for species Ga), snames (names of the states specified for sensitivity analysis), and ifacs (names of the input factors used for sensitivity analysis):

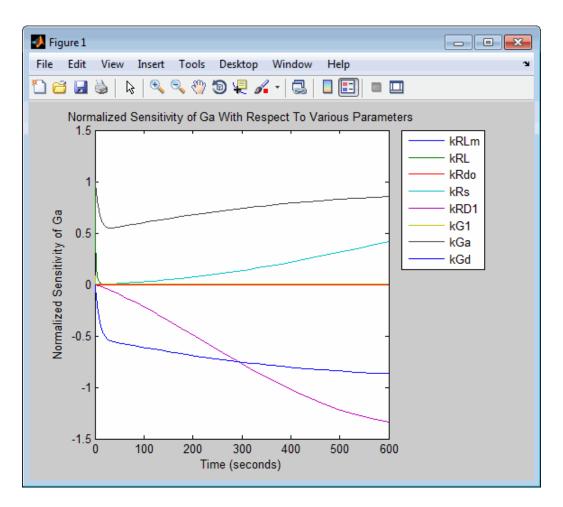
[T, R, snames, ifacs] = getsensmatrix(simDataObj);

**2** Because R is a 3-D array with dimensions corresponding to times, output factors, and input factors, reshape R into columns of input factors to facilitate visualization and plotting:

R2 = squeeze(R);

**3** After extracting the data and reshaping the matrix, plot the data:

```
% Open a new figure
figure;
% Plot time (T) against the reshaped data R2
plot(T,R2);
title('Normalized Sensitivity of Ga With Respect To Various Parameters');
xlabel('Time (seconds)');
ylabel('Normalized Sensitivity of Ga');
% Use the ifacs variable containing the
% names of the input factors for the legend
% Specify legend location and appearance
leg = legend(ifacs, 'Location', 'NorthEastOutside');
set(leg, 'Interpreter', 'none');
```



From the previous plot you can see that Ga is most sensitive to parameters kGd, kRs, kRD1, and kGa. This suggests that the amounts of active G protein in the cell depends on the rate of:

- Receptor synthesis
- Degradation of the receptor-ligand complex
- G protein activation
- G protein inactivation

## **Perform Parameter Scan**

This example shows how to perform a parameter scan by simulating a model multiple times, each time varying the value of a parameter.

In the model described in Model of the Yeast Heterotrimeric G Protein Cycle, the rate of G protein inactivation (kGd) is much lower in the mutant strain versus the wild-type strain (kGd = 0.004 versus kGd = 0.11), which explains higher levels of activated G protein (Ga) in the mutant strain. For a detailed look at how varying the level of kGd affects the level of Ga, perform a parameter scan over five values of kGd.

Load the gprotein.sbproj project, which includes the variable m1, a model object.

sbioloadproject gprotein

The m1 model object appears in the MATLAB Workspace.

View the variants in the m1 model.

#### m1.Variants

SimBiology Variant - mutant (inactive)

ContentIndex:	Type:	Name:	Property:	Value:
1	parameter	kGd	Value	0.004

This model contains one variant named mutant that holds the content for the kGd parameter. This variant is inactive.

Assign the variant to a variable, variantObj, so you can use it to perform the scan.

variantObj = m1.Variants(1);

Create a vector of five evenly spaced values for kGd ranging from 0.001 to 0.15.

```
kGdValues = linspace(1e-3,0.15,5)
```

```
kGdValues =
0.0010 0.0382 0.0755 0.1127 0.1500
```

Initialize a variable, scanData, which you will later use to hold an array of SimData objects to store the results of the parameter scan.

```
scanData = [];
```

Loop over the five kGd values, assigning each to the mutant variant, and specifying the mutant variant to modify the value of kGd when simulating the m1 model. Store the results of the five simulations in an array of SimData objects.

```
for kGd = kGdValues
% Set the value (4th column) of the kGd variant (1st row)
variantObj.Content{1}{4} = kGd;
% Simulate the m1 model using the kGd variant
simDataObj = sbiosimulate(m1,variantObj);
% Store the results in an array of SimData objects
scanData = [scanData;simDataObj];
end
```

The scanData array now contains five SimData objects, with each object containing the data from one simulation in the parameter scan.

Concepts

• "Model of the Yeast Heterotrimeric G Protein Cycle" on page C-19

## **Parameter Estimation**

#### In this section...

"About Parameter Estimation" on page 3-29

"Estimating Parameters of a Model" on page 3-29

"Specifying Solver Type and Options for Parameter Estimation" on page 3-30

"See Also" on page 3-30

"Individual and Population Fitting" on page 3-30

## **About Parameter Estimation**

Parameter estimation lets you estimate the values of unknown parameters in a model by fitting the model simulation results to experimental data. This technique is especially useful for parameters that you do not measure directly. This technique is appropriate when you have a fairly complete data set for one individual.

## **Estimating Parameters of a Model**

You estimate one or more parameters in your model using the sbioparamestim function.

If you do not have Optimization Toolbox<sup>TM</sup> installed, then sbioparamestim uses the MATLAB function fminsearch as the default method for the parameter estimation.

If you have Optimization Toolbox and, optionally, Global Optimization Toolbox installed, then sbioparamestim uses the lsqnonlin function as the default method for the parameter estimation. However, you can specify other optimization functions from these toolboxes as the parameter estimation method.

# Specifying Solver Type and Options for Parameter Estimation

If you specify a stochastic solver and options in the Configset object associated with your model, be aware that during parameter estimation SimBiology temporarily changes:

- SolverType property to the default solver of ode15s
- SolverOptions property to the options last configured for a deterministic solver

## See Also

For more information on parameter estimation, see the sbioparamestim reference page.

For examples of estimating parameters, see:

- "Estimate Parameters" on page 3-32
- Parameter Scanning, Parameter Estimation, and Sensitivity Analysis in the Yeast Heterotrimeric G Protein Cycle

## **Individual and Population Fitting**

SimBiology lets you perform individual and population fitting, which is parameter estimation for grouped sets of experimental data. This functionality requires Statistics Toolbox<sup>™</sup> Version 7.3 or later. This type of parameter estimation is useful for pharmacokinetic/pharmacodynamic (PKPD) models, in which you typically want to fit your model to a population of data. This technique is appropriate when you have an incomplete data set for many individuals.

You perform individual fitting using the sbionlinfit function.

You perform population fitting using the sbionlmefit or sbionlmefitsa function.

For more information, see the following:

• "Creating Pharmacokinetic Models" on page 4-17

- "About Data Fitting in PKPD Models" on page 4-29
- "Performing Data Fitting with PKPD Models" on page 4-34
- Modeling the Population Pharmacokinetics of Phenobarbital in Neonates

## **Estimate Parameters**

#### In this section ...

"Overview" on page 3-32

"Loading the Example Model" on page 3-33

"Defining Experimental Data" on page 3-33

"Simulating the G Protein Model" on page 3-34

"Estimating the kGd Parameter in the G Protein Model" on page 3-36

"Simulating and Plotting Results Using the Estimated Parameter" on page 3-38

"Estimating Other Parameters in the G Protein Model" on page 3-39

## Overview

#### About the Example Model

This example illustrates parameter estimation using time-course data from one experiment, using the sbioparamestim function. For information on all available parameter estimation and population fitting techniques, see "Parameter Estimation" on page 3-29.

This example uses the model described in "Model of the Yeast Heterotrimeric G Protein Cycle" on page C-19 to illustrate parameter estimation.

This table lists the reactions used to model the G protein cycle and the corresponding rate parameters (rate constants) for each mass action reaction. For reversible reactions, the forward rate parameter is listed first.

No.	Name	Reaction <sup>1</sup>	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1

No.	Name	Reaction <sup>1</sup>	Rate Parameters
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1
6	G protein inactivation	Ga -> Gd	kGd

<sup>1</sup> Legend of species: L = ligand (alpha factor), R = alpha-factor receptor, Gd = inactive G-alpha-GDP, Gbg = free levels of G-beta:G-gamma complex, G = inactive Gbg:Gd complex, Ga = active G-alpha-GTP

### About the Example

The study used to build the example model (Yi et al., 2003) reported the estimated value of parameter kGd as 0.11 for the wild-type strain. In "Calculate Sensitivities" on page 3-22, the analysis showed that Ga is sensitive to parameters kGd, kRs, kRD1, and kGa.

This example shows:

- $\bullet\,$  How to estimate the parameter kGd and determine its effect on the model
- $\bullet\,$  How to estimate parameters kGd, kRs, kRD1, and kGa to obtain a better fit to the experimental data

## Loading the Example Model

The gprotein.sbproj project contains a model for the wild-type strain (stored in variable m1). Load the G protein model for the wild-type strain:

```
sbioloadproject gprotein
```

The m1 model object appears in the MATLAB Workspace.

## **Defining Experimental Data**

The study used for this example (Yi et al., 2003) reports, in a plot, the experimental data as the fraction of active G protein. Store the time data for

the experimental results in a variable, tExpt, and store the values for the fraction of active G protein in a variable, GaFracExpt:

```
tExpt = [0 10 30 60 110 210 300 450 600]';
GaFracExpt = [0 0.35 0.4 0.36 0.39 0.33 0.24 0.17 0.2]';
```

**Note** For this simple example, you stored the experimental data in a variable in the MATLAB Workspace by typing the data values. However, typically, you import larger data sets into a MATLAB variable. For more information about importing data into variables, see "Recommended Methods for Importing Data".

## Simulating the G Protein Model

1 Simulate the model and return the results to a SimData object:

```
simDataObj = sbiosimulate(m1);
```

2 Retrieve the time and state data for the GaFrac parameter:

```
[tOrig, GaFracOrig] = selectbyname(simDataObj,'GaFrac');
```

#### Calculating R<sup>2</sup> for the G Protein Model

 $R^2$  is the square of the correlation between the response values and the predicted response values. Therefore,  $R^2$  measures how successful the fit is in explaining the variation of the data.

1 Calculate the sum of squares about the mean (SST):

```
sst = norm(GaFracExpt - mean(GaFracExpt))^2;
```

**2** Interpolate the data to get time points that match the time points in the experimental data using the pchip interpolation method:

```
GaFracResampled = interp1(tOrig, GaFracOrig, tExpt, 'pchip');
```

**3** Calculate the sum of squares due to error (SSE):

```
sse = norm(GaFracExpt - GaFracResampled)^2;
```

**4** Calculate the  $R^2$  value for the simulation data before parameter estimation:

```
rSquareOrig = 1-sse/sst
```

rSquareOrig =

0.8968

For more information about the functions used here, see the norm and interp1 reference pages.

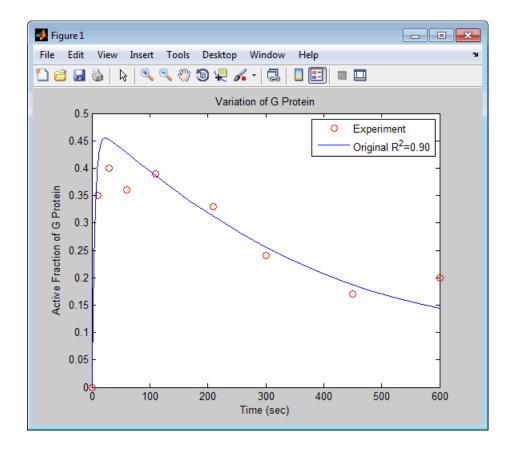
#### **Plotting the Experimental Results and Simulation Data**

**1** Plot the experimental data for active G protein:

```
plot(tExpt, GaFracExpt, 'ro');
title('Variation of G Protein');
xlabel('Time (sec)');
ylabel('Active Fraction of G Protein');
legend('Experiment');
```

**2** Plot the simulation data in the same plot:

```
hold on;
plot(tOrig, GaFracOrig);
legendText = {'Experiment', sprintf('Original R^2=%4.2f',...
rSquareOrig)};
legend(legendText{:});
```



**Note** Leave this figure window open so you can use it to plot and compare results of using the estimated parameters later in this example.

## Estimating the kGd Parameter in the G Protein Model

The study used to build the G protein model reported an estimated value of 0.11 for the parameter kGd in the wild-type strain (Yi et al., 2003). This example estimates the value of kGd.

**1** Create a variable for the parameter to estimate. Also create a variable for the parameter corresponding to the experimental data to which you are fitting:

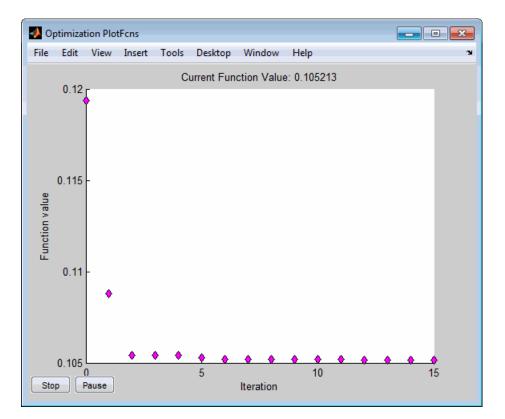
```
paramToEst = sbioselect(m1, 'Name', 'kGd');
GaFrac = sbioselect(m1, 'Name', 'GaFrac');
```

**2** Specify plotting of each iteration of the parameter estimation to see how optimization is progressing:

```
opt = optimset('PlotFcns',@optimplotfval,'MaxIter',15);
```

**3** Use the current value of the kGd parameter in the model as the starting value for optimization:

```
[estValues1, result1] = sbioparamestim(m1, tExpt, GaFracExpt, ...
GaFrac, paramToEst, {}, {'fminsearch',opt});
```



**Note** Close this figure before proceeding with the example.

# Simulating and Plotting Results Using the Estimated Parameter

Use the estimated value of the kGd parameter to see how it affects simulation results.

1 Use a variant to store the estimated value of kGd:

```
estvarObj = addvariant (m1, 'Optimized kGd');
addcontent(estvarObj, {'parameter', 'kGd', 'Value', estValues1});
```

**2** Apply the value stored in the variant, simulate the model, and return the results:

```
simDataObj1 = sbiosimulate(m1, estvarObj );
[t1, GaFrac1] = selectbyname(simDataObj1,'GaFrac');
```

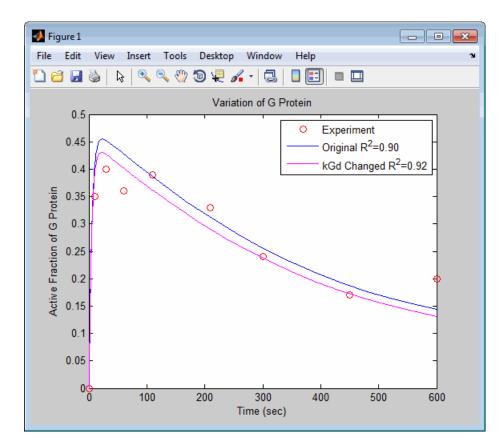
**3** Calculate the  $\mathbb{R}^2$  value with the new estimate obtained using 'fminsearch':

```
GaFrac1Resampled = interp1(t1, GaFrac1, tExpt, 'pchip');
sse1 = norm(GaFracExpt - GaFrac1Resampled)^2;
rSquare1 = 1 - sse1/sst
```

0.9199

**4** Plot the data and compare. If you left the previous figure open, because hold is on, the new plot appears in the existing figure to facilitate the comparison:

```
plot(t1, GaFrac1, 'm-');
legendText{end + 1} = sprintf('kGd Changed R^2=%4.2f', rSquare1);
legend(legendText{:});
```



The figure shows the best fit achieved by changing the parameter kGd.

**Note** Leave this figure window open, so that you can use it later in this example.

## Estimating Other Parameters in the G Protein Model

The example illustrating sensitivity analysis ("Calculate Sensitivities" on page 3-22) showed that Ga is sensitive to parameters kRs, kRD1, kGa, and kGd. Based on the results from the sensitivity analysis, this tutorial shows you how to estimate these parameters. The sensitivity data is presented in "Extracting and Plotting Sensitivity Data" on page 3-24.

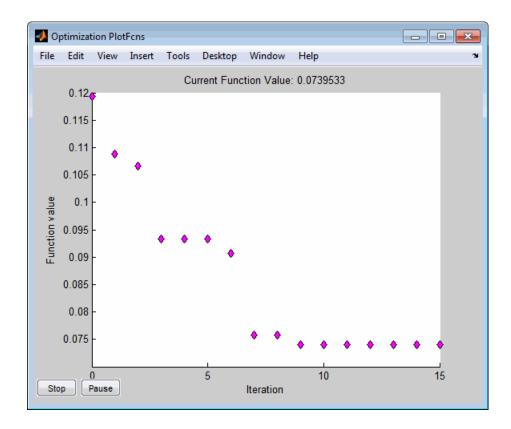
**Note** Although this example estimates four parameters to fit the data, there is no published experimental data that verifies these values, and this example is only for illustration.

1 Create a variable containing the parameters to estimate:

```
paramsToEst = [sbioselect(m1, 'Name', 'kRs');...
    sbioselect(m1, 'Name', 'kRD1');...
    sbioselect(m1, 'Name', 'kGa');...
    sbioselect(m1, 'Name', 'kGd')];
```

2 Estimate the parameters. Use the current values of parameters in the model as the starting values for optimization. Use the opt variable you created previously to specify plotting of each iteration of the parameter estimation to see how optimization is progressing:

```
[estValues2, result2] = sbioparamestim(m1, tExpt, GaFracExpt,...
GaFrac, paramsToEst, {}, {'fminsearch',opt});
```



**Note** Close this figure before proceeding with the example.

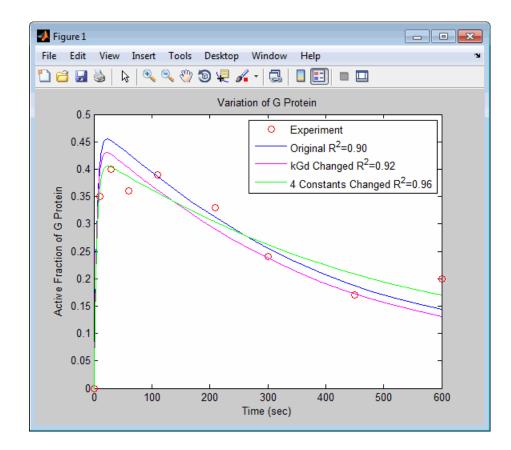
**3** Compare original parameter values and the estimated parameter values obtained with 'fminsearch':

% Origina: paramsToEs	•	er values	
SimBiology	/ Paramete	er Array	
Index: 1	Name: kRs	Value: 4	ValueUnits:
2	kRD1	0.004	

```
kGa
  3
                       1e-005
  4
             kGd
                       0.11
  % Estimated parameter values
  num2str(estValues2)
  ans =
        4.549
    0.0031018
  9.0068e-006
      0.12381
4 Calculate the \mathbb{R}^2 value using the new estimates obtained with
  'fminsearch':
  estvarObj2 = addvariant(m1, 'Optimized kRs, kRD1, kGa, and kGd');
  addcontent(estvarObj2, ...
      {{'parameter', 'kRs', 'Value', estValues2(1)}, ...
      {'parameter', 'kRD1', 'Value', estValues2(2)}, ...
{'parameter', 'kGa', 'Value', estValues2(3)}, ...
      {'parameter', 'kGd', 'Value', estValues2(4)}});
  simDataObj2 = sbiosimulate(m1, estvarObj2);
  [t2, GaFrac2] = selectbyname(simDataObj2, 'GaFrac');
  GaFrac2Resampled = interp1(t2, GaFrac2, tExpt, 'pchip');
  sse2 = norm(GaFracExpt - GaFrac2Resampled)^2;
  rSquare2 = 1 - sse2/sst
  rSquare2 =
      0.9603
5 Plot the data and compare. If you left the previous figure open, because
```

**5** Plot the data and compare. If you left the previous figure open, because hold is on, the new plot appears in the existing figure to facilitate the comparison:

```
plot(t2, GaFrac2, 'g-');
legendText{end + 1} = sprintf('4 Constants Changed R^2=%4.2f',...
rSquare2);
legend(legendText{:});
```



# **Accelerating Model Simulations and Analyses**

#### In this section ...

"What Is Acceleration?" on page 3-44

"What Simulations and Analyses Can Be Accelerated?" on page 3-44

"When to Accelerate Simulations and Analyses" on page 3-45

"Prerequisites for Accelerating Simulations and Analyses" on page 3-45  $\,$ 

"Accelerate a Simulation or Analysis" on page 3-45

"Troubleshooting Accelerated Simulations and Analyses" on page 3-46

## What Is Acceleration?

Normally, when simulating or analyzing a model in SimBiology, you express the model in MATLAB code. You can accelerate the simulation or analysis by converting the model to compiled C code, which executes faster. Because this compilation step has a small time overhead, acceleration is not recommended for individual simulations of small models. However, for large models, or for repeated simulations during analysis, acceleration can provide a significant speed increase that outweighs the small time overhead.

## What Simulations and Analyses Can Be Accelerated?

You can accelerate the following:

- Simulating models
- Calculating sensitivities

**Note** For parameter estimations (using sbioparamestim) and population fittings (using sbionlinfit, sbionlmefit, or sbionlmefitsa), acceleration is automatically enabled, if the prerequisites for accelerating simulations and analyses are met.

## When to Accelerate Simulations and Analyses

The functionality to accelerate simulations performs optimally under the following conditions:

- Running many simulations with different initial conditions
- Running very long simulations (for example, simulations that take longer than a minute to run)

# Prerequisites for Accelerating Simulations and Analyses

To prepare your models for accelerated simulations, install and set up a compiler:

1 Install a C compiler (if one is not already installed on your system). For a current list of supported compilers, see Supported and Compatible Compilers at www.mathworks.com.

**Tip** On 32-bit Windows<sup>®</sup> platforms, the lcc compiler is automatically installed. However, for better performance of the acceleration functionality, you may want to install a supported compiler other than lcc. You can also use mex -setup to choose and configure a different C compiler, as described in "Build MEX-Files" in the MATLAB External Interfaces documentation.

- **2** Run mex -setup once after compiler installation, as described in the mex reference page.
- **3** Ensure that any user-defined functions in your model can be used for code generation from MATLAB, so they can convert to compiled C. For more information, see the "Code Generation from MATLAB Code".

## Accelerate a Simulation or Analysis

Accelerating simulations is a two step process:

1 Use the sbioaccelerate function to prepare your model for accelerated simulations. Use the same input arguments that you plan to use with sbiosimulate. For example:

```
sbioaccelerate(modelObj, configsetObj, doseObj);
```

This step prepares your model for acceleration and may take a minute or longer to complete for very large models.

**Note** You need to run sbioaccelerate again, before running simulations, if you make any modifications to this model, other than:

- Changes to any variants
- Changes to values for the InitialAmount property of species
- Changes to the Capacity property of compartments
- Changes to the Value property of parameters
- **2** Use the sbiosimulate function with the same input arguments that you used with sbioaccelerate. For example:

simdataObj = sbiosimulate(modelObj, configsetObj, doseObj);

# Troubleshooting Accelerated Simulations and Analyses

If you have user-defined functions, do not use persistent variables in these functions. Persistent variables are not compatible with the functionality used for accelerating simulations.

If you specify user-defined functions in SimBiology expressions, you might see the following warning if your code is not compatible with code generation from MATLAB:

The SimBiology *Expression* and any user-defined functions could not be accelerated. Please check that these expressions and any user-defined functions are supported for code generation as described in the Code Generation from MATLAB documentation.

where *Expression* is any of the following:

• Reaction rate/rule expression

- Repeated assignment rule expression
- Event trigger expression
- Event function expression

For more information, see "Code Generation".

# Pharmacokinetic Modeling

- "Pharmacokinetic Modeling Functionality" on page 4-2
- "Importing Data Supported Files and Data Types" on page 4-8
- "Importing Data" on page 4-13
- "Creating Pharmacokinetic Models" on page 4-17
- "About Data Fitting in PKPD Models" on page 4-29
- "Performing Data Fitting with PKPD Models" on page 4-34
- "PK/PD Modeling and Simulation to Guide Dosing Strategy for Antibiotics" on page 4-54
- "Deploy a SimBiology Model" on page 4-68

## **Pharmacokinetic Modeling Functionality**

#### In this section...

"Overview" on page 4-2

"Required and Recommended Software for Pharmacokinetic Modeling" on page 4-3

"How SimBiology Supports Pharmacokinetic Modeling" on page 4-4

"Using the Command Line Versus the SimBiology Desktop" on page 4-5

"Pharmacokinetic Modeling Example" on page 4-6

```
"Acknowledgements: Tobramycin Data Set" on page 4-6
```

## Overview

SimBiology software extends the MATLAB computing environment for analyzing pharmacokinetic (PK) data using models. The software lets you do the following:

- Create models Use a model construction wizard. Alternatively, extend any model with pharmacodynamic (PD) model components, or build higher fidelity models. See "Model" on page 4-4 for more information.
- Fit data Fit nonlinear, mixed-effects models to data, and estimate the fixed and random effects, or fit the data using nonlinear least squares. For more information, see "Analyze Data Using Models" on page 4-4.
- Generate diagnostic plots For more information, see "Analyze Data Using Models" on page 4-4.

The software lets you work with different model structures, thus letting you try multiple models to see which one produces the best results.

# Required and Recommended Software for Pharmacokinetic Modeling

#### **Required Software**

MATLAB	Provides a command-line interface and an integrated software environment. For instructions, see the MATLAB installation documentation for your platform.	
	If you have installed MATLAB and want to check which other MathWorks <sup>®</sup> products are installed, enter ver in the MATLAB Command Window.	
<b>Statistics Toolbox</b> (Version 7.3 (R2010a) or greater)	Provides fitting tools including functions used to analyze nonlinear mixed effects.	

#### **Recommended Software**

C Compiler	Required to prepare the model for accelerating simulations. For list of supported compilers, see Supported and Compatible Compilers.
Optimization Toolbox	Optimization Toolbox extends the MATLAB technical computing environment with tools and widely used algorithms for standard and large-scale optimization. These algorithms solve constrained and unconstrained, continuous and discrete problems. If the Optimization Toolbox product is installed, you can specify additional methods for likelihood maximization. If you do not have this product, SimBiology

uses fminsearch provided by MATLAB for likelihood maximization.

## How SimBiology Supports Pharmacokinetic Modeling

#### Import and Work with Data

You can import tabular data into the SimBiology desktop or the MATLAB Workspace. The supported file types are .xls, .csv, and .txt. You can specify that the data is in a NONMEM<sup>®</sup> formatted file. The import process interprets the columns according to the NONMEM definitions.

From the SimBiology desktop, you can filter the raw data to suppress outliers, visualize data using common plots (such as plot, semilog, scatter, or stairs), and perform basic statistical analysis. You also can use functions to process and visualize the data at the command line.

#### Model

SimBiology provides an extensible modeling environment. You can do any of the following:

- Create a PK model using a model construction wizard to specify the number of compartments, the route of administration, and the type of elimination.
- Extend any model with pharmacodynamic (PD) model components, or build higher fidelity models.
- Build or load your own SimBiology, or SBML model.

For more information on building SimBiology models, see "What is a Model?" on page 1-2.

#### **Analyze Data Using Models**

Perform both individual and population fits to grouped longitudinal data:

• Individual fit — Fit data using nonlinear least-squares method, specify parameter transformations, estimate parameters, and calculate residuals and the estimated coefficient covariance matrix.

• Population fit — Fit data, specify parameter transformations, and estimate the fixed effects and the random sources of variation on parameters using nonlinear mixed-effects models.

You can use the following methods to estimate the fixed effects:

- LME Linear mixed-effects approximation
- RELME Restricted LME approximation
- F0 First-order estimate
- FOCE First-order conditional estimate

For more information about each of these methods, see nlmefit in the Statistics Toolbox documentation.

• Population fit using a stochastic algorithm — Fit data, specify parameter transformations, and estimate the fixed effects and the random sources of variation on parameters, using the Stochastic Approximation Expectation-Maximization (SAEM) algorithm. SAEM is more robust with respect to starting values. This functionality relaxes assumption of constant error variance.

For more information, see nlmefitsa in the Statistics Toolbox documentation.

In addition, you can generate diagnostic plots that show:

- The predicted time courses and observations for an individual or the population
- Observed versus predicted values
- Residuals versus time, group, or predictions
- Distribution of the residuals
- A box-plot for random effects or parameter estimates from individual fitting

## Using the Command Line Versus the SimBiology Desktop

SimBiology extends MATLAB and lets you access pharmacokinetic modeling functionality at the command line and in the graphical SimBiology desktop.

Use the command line to write and save scripts for batch processing and to automate your workflow.

Use the SimBiology desktop to interactively change and iterate through the model workflow. The SimBiology desktop lets you encapsulate models, data, tasks, task settings, and diagnostic plots into one convenient package, namely a SimBiology project.

Furthermore, if you are using the SimBiology desktop and want to learn about using the command line, the MATLAB code capture feature in the desktop lets you see the commands and export files for further scripting in the MATLAB editor.

#### Pharmacokinetic Modeling Example

For an example showing pharmacokinetic modeling functionality at the command line, see Modeling the Population Pharmacokinetics of Phenobarbital in Neonates.

#### Acknowledgements: Tobramycin Data Set

Acknowledgements for data in the tobramycin.txt file in the /matlab/toolbox/simbio/simbiodemos folder:

## References

[1] Original Publication: Aarons L, Vozeh S, Wenk M, Weiss P, and Follath F. "Population pharmacokinetics of tobramycin." *Br J Clin Pharmacol.* 1989 Sep;28(3):305–14.

Data set provided by Dr. Leon Aarons, (laarons@fs1.pa.man.ac.uk)

The data in the tobramycin.txt file were downloaded from the Web site of the Resource Facility for Population Kinetics http://depts.washington.edu/rfpk/service/datasets/index.html (no longer active). Funding source: NIH/NIBIB grant P41-EB01975. The original data set was modified as follows:

- Header comments were removed.
- The file was converted to a tab-delimited format.
- Missing values in the HT column were denoted with "." instead of 10000000.000.

## Importing Data – Supported Files and Data Types

#### In this section ...

"Supported Files and Data Types" on page 4-8

"Support for Importing NONMEM Formatted Files" on page 4-8

"Creating a Data File with SimBiology Definitions" on page 4-12

#### **Supported Files and Data Types**

You can import tabular data to the SimBiology desktop or to the MATLAB Workspace. The supported file types are .xls, .csv, and .txt. You can specify that the data is in a NONMEM formatted file. The import process interprets the columns according to the NONMEM definitions. For more information see "Support for Importing NONMEM Formatted Files" on page 4-8.

From the SimBiology desktop, you can filter the raw data to suppress outliers, visualize data using common plots (such as plot, semilog, scatter, or stairs), and perform basic statistical analysis. You also can use functions to process and visualize the data at the command line.

**Note** If your data set contains dosing information that is infusion data, the data set must contain the rate and not an infusion duration.

#### **Unit Conversion**

Regardless of whether unit conversion functionality is on or off, dosing in the data file must be expressed in amounts (or as amount/time for infusion rate). By default **Unit Conversion** is off, so you must ensure that units for the data are consistent with each other. If you want to turn on unit conversion, see "Unit Conversion for Imported Data" on page 4-27.

### **Support for Importing NONMEM Formatted Files**

You can specify that the data is in a NONMEM formatted file. The following table highlights the interpretation of this data in SimBiology software.

Column Header	Interpretation
ID	Text or numeric values that identify the record. The import process assumes that contiguous data with the same value contains data from one individual. If the data contains non-contiguous references to the same value, the import process assigns the second ID encountered an indexed valued derived from the group first encountered. For example, if the ID columns contains [1 1 1 2 2 2 1 1 1], the IDs assigned are 1, 2, 1_1.
TIME	Monotonically increasing positive values within each group, indicating time of observation or dose. The data file can specify clock (2:30) or decimal values (6.25). The import process assigns a value of 0 to the first TIME value in the data file. The import process assigns subsequent values relative to the first value. For example the import process interprets [10:05 10:30 11 12:30 21.3] as: [0 0.25 0.95 2.25 14.2].
	If the data file also contains a DATE column, the import process uses it with the TIME column in calculating the relative TIME values. The column cannot contain Inf.
DATE, DAT1, DAT2, or DAT3	Defines the day of the observation or the dose. The column can contain the month as a number (9) or a string (Sep). Specify date in the following formats:
	• DATE — The column can specify month/day/year or month-day-year. If you specify two numbers, the import process assumes they are month and day.
	• DAT1 — The column can specify day/month/year or day-month-year. If you specify two numbers, the import process assumes they are day and month.

Column Header	Interpretation
	<ul> <li>DAT2 — The column can specify year/month/day or year-month-day. If you specify two numbers, the import process assumes they are month and day.</li> <li>DAT3 — The column can specify year/day/month or year-day-month. If you specify two numbers, the import process assumes they are day and month.</li> <li>Note the following additional assumptions:</li> <li>If you specify only one number, the import process assumes it is the day</li> </ul>
	• You can omit the year or specify 1, 2, 3, or 4 digits. If you specify two-digit years, it is assumed to be in the 1900s.
DV	Numeric value of an observation. Column cannot contain Inf or Inf.
MDV	<ul><li>Defines whether a row describes an observation:</li><li>Row contains 0 — Observation event</li></ul>
	• Row contains 1 — Not an observation event
EVID	<ul> <li>Defines the type of event described for the row in the record:</li> <li>0 — Observation event; row contains an observed value.</li> </ul>
	• 1 — Dose event; row describes a dose.
	• 2 — Other event; row describes some other event such as measurement of a covariate.
	If a column contains values for dose, but EVID is not 1, the import process ignores the value. You see a warning and the value is ignored.

Column Header	Interpretation
	The import process does not support values <b>3</b> and <b>4</b> . You see a warning and the value is ignored.
СМТ	Indicates which compartment is used for observation value or for dose received. The interpretation also depends on EVID:
	• Observation event (EVID = 0) — CMT column indicates which compartment was used for observation value.
	• Dose Event (EVID = 1) — CMT column indicates which compartment received the dose.
AMT	Positive number indicating dose. O or NaN specifies no dose administered. The column cannot contain Inf.
RATE	Positive number indicating rate of infusion. 0 specifies an infinite rate (equivalent to a bolus dose), and NaN specifies no rate. The column cannot contain Inf.
II	Positive number defining the time between doses.
ADDL	When the data specifies a number of identical serial doses at specific intervals (defined by II), ADDL specifies the number of doses in the series excluding the initial dose. If the data specifies II but not ADDL, then SimBiology assumes that the dosing occurs for the duration of that data record.

#### **Unsupported NONMEM Definitions**

The import process does not support (and therefore ignores) the rows containing the following values or definitions:

- $\bullet~{\sf EVID}$  values  $3~{\rm and}~4$
- SS column for specifying steady state doses

- PCMT column to define whether to compute a prediction for the row
- CALL column for calling the ERROR or the PK subroutine
- If rate is specified as being less than zero, it is assumed to be zero

## **Creating a Data File with SimBiology Definitions**

If you are creating a file containing population data that you want to later import into SimBiology, create the data file with the following columns:

- Group column Specify text or numeric values. The rows in the file that have the same Group column value are for the same individual.
- Time column Specify monotonically increasing positive values within each group that define the time of the dose, observation and/or covariate measurements.
- Zero or more dosing columns Create one dosing column for each compartment being dosed. In each column, specify positive values representing doses in amount that are added to a species. Use 0 or NaN to specify that no dose was applied at the specified time. This is useful for times when an observation was recorded but no dose was applied.
- Zero, or more rate columns Specify positive values. Zero specifies an infinite rate and NaN specifies that no rate applies. The rate column is associated with a dosing column and defines the rate at which the dose is administered.
- Zero or more observation columns Specify numeric values or NaNs. You can only specify one observation value at a particular time for each group. NaN values define that no observation was recorded at the specified time. This is useful for times when a dose was applied but no observation was recorded.
- Zero or more covariate columns Specify numeric values or NaNs. Each value defines the covariate value at the specified time. NaN values define that no covariate observation was recorded at the specified time.

## **Importing Data**

#### In this section ...

"Importing Data from Files" on page 4-13

"Importing Data from NONMEM Formatted Files" on page 4-14

"Other Resources for Importing Data" on page 4-15

### **Importing Data from Files**

Use the dataset function to import tabular data with named columns into an array that you can use in fitting and analysis at the command line. Use this function when you want to import the data without NONMEM interpretation of column headers. The dataset function lets you specify parameter/value pair arguments in which you can specify options such as the type of delimiter, and whether the first row contains header names. For more information, see dataset.

To prepare the data file for import, remove any comments that are present at the beginning of the file.

Examples:

```
% text files
data = dataset('file', 'tobramycin.txt')
% text files with . in place of missing values
data = dataset('file', 'tobramycin.txt', 'TreatAsEmpty', '.')
% For Excel files
data = dataset('xlsfile', 'tobramycin.xls')
```

You can also construct the dataset array from variables in the MATLAB Workspace.

```
% Create a 10x2 array
x = rand(10,2);
% Construct a dataset array containing x
data = dataset({x(:, 1), 'Column1'}, {x(:,2), 'Column2'})
```

If you import the data as separate variables containing doubles, you can construct the dataset array by concatenating the variables.

```
% Create 2 10x1 vectors
x = rand(10,1);
y = rand(10,1);
% Construct a dataset array containing x and y
data = dataset({x, 'Column1'}, {y, 'Column2'})
```

After you finish analyzing your data, you can export any new variables in the MATLAB Workspace to a variety of file formats.

#### **Importing Data from NONMEM Formatted Files**

Use the sbionmimport function to import data from NONMEM formatted files. To import the data without NONMEM interpretation of column headers, see "Importing Data from Files" on page 4-13.

To prepare the data file for import, remove any comments that are present at the beginning of the file and select one of the following methods to import your data:

• If the data file contains only the column header values shown in "Support for Importing NONMEM Formatted Files" on page 4-8, use the syntax shown in the following example:

```
filename = 'C:\work\datafiles\dose.xls';
ds = sbionmimport(filename);
```

- If the data file has column header labels different from the table shown in "Support for Importing NONMEM Formatted Files" on page 4-8 and you want to apply NONMEM interpretation of headers:
  - 1 Create a NONMEM file definition object. This object lets you define what the column headers in the data file mean in SimBiology. In the following example, the column containing response values is CP, whereas in NONMEM formatted files the column is labelled DV.

To use the tobramycin data set [1], create a NONMEM file definition object and define the following:

```
def = sbionmfiledef;
def.DoseLabel = 'DOSE';
def.GroupLabel = 'ID';
def.TimeLabel = 'TIME';
def.DependentVariableLabel = 'CP';
def.MissingDependentVariableLabel = 'MDV';
def.EventIDLabel = 'EVID';
def.ContinuousCovariateLabels = {'WT', 'HT', 'AGE', 'SEX', 'CLCR'};
```

Your file can contain any name for column headings. See sbionmfiledef for the list of properties you can configure in the NONMEM file definition object.

2 Use the sbionmimport function to import your data file with the column header definitions as specified in the NONMEM file definition object. For example, browse to matlabroot/toolbox/simbio/simbiodemos/ (where matlabroot is the folder where MATLAB is installed).

```
[data, pkDataObject] = sbionmimport('tobramycin.txt', def, ...
'TreatAsEmpty', '.');
```

This example shows you how to obtain the PKData object, PKDataObj, while importing, since you will use the PKData object in fitting the model later.

The sbionmimport function accepts property-name-value pairs accepted by dataset. For example, if the data set does not contain column headers, use 'ReadVarNames', false to specify that sbionmimport should read values from the first row of the file.

For information about creating a model to fit the data, see "Creating Pharmacokinetic Model Using the Command Line" on page 4-19.

#### **Other Resources for Importing Data**

For detailed information about supported data formats and the functions for importing data into the MATLAB Workspace, see the "Recommended Methods for Importing Data". You also can import data using the MATLAB Import Wizard (see "Tips for Using the Import Wizard". Use the Import Wizard, to import data as text files (such as .txt and .dat), MAT-files, and spreadsheet files, (such as .xls).

The MATLAB Import Wizard processes the data source. The wizard recognizes data delimiters, as well as row or column headers, to facilitate the process of data selection and importation into the MATLAB Workspace. You can import the data to the SimBiology desktop from the MATLAB Workspace.

## **Creating Pharmacokinetic Models**

#### In this section ...

"Ways to Create or Import Pharmacokinetic Model" on page 4-17

"How SimBiology Models Represent Pharmacokinetic Models" on page 4-17

"Creating Pharmacokinetic Model Using the Command Line" on page 4-19

"Dosing Types" on page 4-21

"Elimination Types" on page 4-24

"Intercompartmental Clearance" on page 4-26

"Unit Conversion for Imported Data" on page 4-27

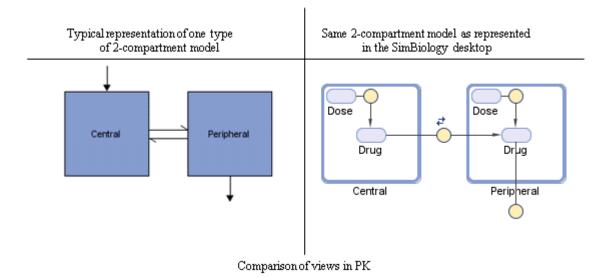
### Ways to Create or Import Pharmacokinetic Model

To start modeling, you can:

- Create a PK model using a model construction wizard that lets you specify the number of compartments, the route of administration, and the type of elimination.
- Extend any model to build higher fidelity models.
- Build or load your own model. Load a SimBiology project or SBML model.

#### How SimBiology Models Represent Pharmacokinetic Models

The following figure compares a model as typically represented in pharmacokinetics with the same model shown in the SimBiology model diagram. For this comparison, assume that you are modeling administration of a drug using a two-compartment model with any dosing input and linear elimination kinetics. (The model structure remains the same with any dosing type.)



Note the following:

- SimBiology represents the concentration or amount of a drug in a given compartment or volume by a species *object* contained within the compartment.
- SimBiology represents the exchange or flow of the drug between compartments and the elimination of the drug by reactions.
- SimBiology represents intercompartmental clearance by a parameter (Q) which specifies the clearance between the compartments.
- SimBiology drives the dosing schedule with a combination of species (Drug and/or Dose) and reactions (Dose -> Drug), depending on whether the administration into the compartment follows bolus, zero-order, infusion, or first-order dosing kinetics. For more information on the components added and parameters estimated, see "Dosing Types" on page 4-21.

You can also view this model as a regression function, y = f(k, u), where y is the predicted value, given values of an input u, and parameter values k. In SimBiology the model represents f, and the model is used to generate a regression function if y, k, and u are identified in the model.

## Creating Pharmacokinetic Model Using the Command Line

To create a PK model with the specified number of compartments, dosing type, and method of elimination:

1 Create a PKModelDesign object. The PKModelDesign object lets you specify the number of compartments, route of administration, and method of elimination, which SimBiology uses to construct the model object with the necessary compartments, species, reactions, and rules.

```
pkm = PKModelDesign;
```

2 Add a compartment specifying the compartment name, and optionally, the type of dosing, and the method of elimination. Also specify whether the data contains a response variable measured in this compartment and whether the dose(s) have time lags. For example, if using the tobramycin data set [1], specify a compartment named Central, with Bolus for the DosingType property, linear-clearance for the EliminationType property, and true for the HasResponseVariable property.

```
pkc1 = addCompartment(pkm, 'Central', 'DosingType', 'Bolus', ...
'EliminationType', 'linear-clearance', ...
'HasResponseVariable', true);
```

For a description of other DosingType and EliminationType property values, see "Dosing Types" on page 4-21 and "Elimination Types" on page 4-24.

For a description of the HasResponseVariable property, see HasResponseVariable. At least one compartment in a model must have a response. Although SimBiology supports multiple responses per compartment, when adding compartments to a PKModelDesign object, you are limited to one response per compartment. **Note** To add a compartment that has a time lag associated with any dose that targets it, set the HasLag property to true:

```
pkc_lag = addCompartment(pkm, 'Central', 'DosingType', 'Bolus', ...
'EliminationType', 'linear-clearance', ...
'HasResponseVariable', true, 'HasLag', true);
```

Or after adding a compartment, set its HasLag property to true:

```
pkc1.HasLag = true;
```

**3** Optionally, add a second compartment named Peripheral, with no dosing, no elimination, and no time lag. Set the HasResponseVariable property to true. If you are using the tobramycin data set [1], skip this step and use only one compartment.

pkc2 = addCompartment(pkm, 'Peripheral', 'HasResponseVariable', true);

The model construction process adds the necessary parameters, including a parameter representing intercompartmental clearance Q. You can add more compartments by repeating this step. The addition of each compartment creates a chain of compartments in the order of compartment addition, with a bidirectional flow of the drug between compartments in the model.

Use the handle to the compartment (pkc1 or pkc2), to change compartment properties.

4 Construct a SimBiology model object.

```
[modelObj, PKModelMapObj] = pkm.construct
```

The construct method returns a SimBiology model object (modelObj) and a PKModelMap object (PKModelMapObj) that contains the mapping of the model components to the elements of the regression function. For more information about the PKModelMap object, see "Defining Model Components for Observed Response, Dose, Dosing Type, and Estimated Parameters" on page 4-31. **Note** If you change the PKModelDesign object, you must create a new model object using the construct method. Changes to the PKModelDesign do not automatically propagate to a previously constructed model object.

**5** Perform parameter fitting as shown in "Performing Data Fitting with PKPD Models" on page 4-34.

The model object and the PKModelMap object are input arguments for the sbionlmefit, sbionlmefitsa and sbionlinfit functions used in parameter fitting.

For information on	See
Dosing types	"Dosing Types" on page 4-21
Elimination types	"Elimination Types" on page 4-24
Parameter fitting	"Performing Data Fitting with PKPD Models" on page 4-34
Simulating the model and a description of configuration sets	"Model Simulation" on page 3-3

### **Dosing Types**

When creating models, SimBiology creates the following model components for each compartment in the model, regardless of the dosing type:

- Two species (Drug\_CompartmentName and Dose\_CompartmentName) for each compartment.
- A reaction (Dose\_CompartmentName -> Drug\_CompartmentName) for each compartment, governed by mass action kinetics.
- A parameter (ka\_CompartmentName) for each compartment, representing the absorption rate of the drug when absorption follows first-order kinetics. This is the forward rate parameter for the Dose\_CompartmentName -> Drug\_CompartmentName reaction.

- A parameter (Tk0\_CompartmentName) for each compartment, representing the duration of drug absorption when absorption follows zero-order kinetics.
- A parameter (TLag\_CompartmentName) for each compartment, representing the time lag for any dose that targets that compartment and also that is specified as having a time lag.

For dosing types that have a fixed infusion or absorption duration (infusion and zero-order), you can use overlapping doses. The doses are additive.

Dosing Type	Description	SimBiology Model Components Used	Default Parameters to Estimate
''(empty string)	No dose	The species (Drug_CompartmentName) in each compartment	None
SimBiology desktop — bolus Command line — Bolus	Assumes that the drug amount is increased instantly at the dose time. In the SimBiology model, the initial concentration of the drug is based on dose amount and volume of the compartment containing the drug.	The species (Drug_CompartmentName) in each compartment	None

The following table describes the dosing types, the default parameters to estimate, and lists the model components created and used for dosing.

Dosing Type	Description	SimBiology Model Components Used	Default Parameters to Estimate
SimBiology desktop — infusion Command line — Infusion	Assumes that the infused drug amount increases at a constant known absorption (or infusion) rate over a known duration. The imported data set must contain the rate and not an infusion duration. SimBiology uses this information to change the species concentration at the constant rate over the duration specified in the data set.	The species (Drug_CompartmentName) in each compartment	None
SimBiology desktop — zero-order Command line — ZeroOrder	Assumes that the drug is added at a constant rate over fixed, but unknown duration.	<ul> <li>The species Drug_CompartmentName in each compartment</li> <li>The parameter (Tk0_CompartmentName) in each compartment that has zero-order dosing. This parameter represents the duration of drug absorption</li> </ul>	Tk0_CompartmentName (absorption duration)
SimBiology desktop — first-order Command line — FirstOrder	Assumes that the rate at which the drug is absorbed is not constant. In the SimBiology model, absorption rate is assumed	<ul> <li>A species         <ul> <li>(Dose_CompartmentName) representing the dose amount before it is absorbed</li> </ul> </li> <li>A species         <ul> <li>(Drug_CompartmentName) for each compartment</li> </ul> </li> </ul>	ka_CompartmentName (absorption rate)

Dosing Type	Description	SimBiology Model Components Used	Default Parameters to Estimate
	to be governed by mass-action kinetics.	<ul> <li>A parameter (ka_CompartmentName) representing the absorption rate of the drug</li> <li>A MassAction reaction (Dose_CompartmentName) &gt; Drug_CompartmentName) with forward rate parameter (ka_CompartmentName)</li> </ul>	

If you are using a custom model, or want to simulate a model with the dosing schedule applied, see the following additional sources of information:

For information on	See
Preparing the model before simulating	"Prerequisites for Using Custom SimBiology Models in Data Fitting" on page 4-30

## **Elimination Types**

Elimination Type	Description	SimBiology Model Components Created	Default Parameters to Estimate
SimBiology desktop — Linear {Elimination Rate, Volume} Command line — 'linear'	Assumes simple mass-action kinetics in the elimination of the drug. In the SimBiology model, elimination is specified by mass-action kinetics with the	<ul> <li>A parameter representing the elimination rate (ke_CompartmentName)</li> <li>A MassAction reaction (drug &gt; null) with forward rate parameter (ke_CompartmentName)</li> </ul>	<ul> <li>Compartment volume         (Capacity property)</li> <li>Elimination rate constant (ke_CompartmentName)</li> <li>Inter-compartmental clearance (Q) when</li> </ul>

Elimination Type	Description	SimBiology Model Components Created	Default Parameters to Estimate
SimBiology	elimination rate constant specified by the forward rate parameter (ke). Assumes simple	<ul> <li>specific to the compartement</li> <li>A parameter</li> </ul>	<ul> <li>there is more than one compartment.</li> <li>See "Intercompartmental Clearance" on page 4-26.</li> <li>Compartment</li> </ul>
desktop — Linear {Clearance, Volume} Command line — 'linear-clearance'	<pre>mass-action kinetics in the elimination of the drug. In the SimBiology model, similar to Linear {Elimination Rate, Volume}. But, in addition, this option lets you specify the model in terms of clearance (C1) where, C1 = ke * volume).</pre>	<pre>representing the clearance (C1_CompartmentName) • A parameter representing the elimination rate constant (ke_CompartmentName) • An InitialAssignment rule that initializes ke_CompartmentName based on the initial values for C1_CompartmentName and compartment volume • A MassAction reaction (drug &gt; null) with forward rate parameter (ke_CompartmentName)</pre>	<ul> <li>volume (Capacity property)</li> <li>Clearance (C1_CompartmentName)</li> <li>Inter-compartmental clearance (Q) when there is more than one compartment. See "Intercompartmental Clearance" on page 4-26.</li> </ul>
SimBiology desktop — Enzymatic (Michaelis-Menten)	Assumes that elimination is governed by	Parameter representing the	<ul> <li>Compartment volume (Capacity property)</li> </ul>

Elimination Type	Description	SimBiology Model Components Created	Default Parameters to Estimate
Command line — 'enzymatic'	Michaelis-Menten kinetics.	<ul> <li>Michaelis constant, (Km_CompartmentName)</li> <li>A parameter for maximum velocity (Vm_CompartmentName</li> <li>A reaction with Michaelis-Menten kinetics (drug -&gt; null), with kinetic law parameters Vm_CompartmentName and Km_CompartmentName</li> </ul>	<ul> <li>Parameter (Km_CompartmentName)</li> <li>Parameter (Vm_CompartmentName)</li> <li>Inter-compartmental clearance (Q) when there is more than one compartment. See "Intercompartmental Clearance" on page 4-26</li> </ul>

## Intercompartmental Clearance

The compartments created when you generate a SimBiology model form a chain and each pair of linked compartments are connected by a transport reaction similar to linear elimination. The addition of two compartments, C1 and C2, generates a reversible mass-action reaction C1.Drug\_C1 <-> C2.Drug. The forward rate parameter is the compartmental clearance,  $Q_{12}$ , divided by the volume of C1. The reverse rate parameter is  $Q_{12}$ , divided by the volume of C2.

The process of adding each pair of compartments in the chain  ${\tt C_m}$  and  ${\tt C_n}$  generates the following model components:

- A parameter Q<sub>mn</sub> representing the compartmental clearance between those two compartments. This parameter is added to the list of parameters to be estimated (PKModelMapObj.Estimated property).
- A parameter (kmn) representing the rate of transfer of the drug from Cm to Cn, where  $k_{mn}$  =  $Q_{mn}/V_m$ .
- A parameter (knm) representing the rate of Cn to Cm, where  $k_{nm} = Q_{mn}/V_n$ .

- A reversible mass-action reaction between the two compartments, Cm.Drug\_Cm <-> Cn.Drug\_Cn, with forward rate parameter kmn, and reverse rate parameter knm.
- An initial assignment rule that initializes the value of the parameter kmn, based on the initial values for Cm and Qmn.
- An initial assignment rule that initializes the value of the parameter knm, based on the initial values for Cn and Qmn.

#### **Unit Conversion for Imported Data**

Unit conversion converts the matching physical quantities to one consistent unit system in order to resolve them. This conversion is in preparation for correct simulation, but SimBiology returns the physical quantities in the model in units that you specify.

Regardless of whether unit conversion is on or off, you must express dosing data in amount. By default, **Unit Conversion** is off, so you must ensure that units for the data and the model are consistent with one another.

If **Unit Conversion** is on, you must specify units. If using the SimBiology desktop, specify units in the **Raw Data** tab, when data is selected in the **Project Explorer**. If using the command line, specify units in the PKData object.

Parameters in the model have default units. If unit conversion is on, you can change the units as long as the dimensions are consistent. These default units, which you might use to specify the values for the initial guess, are as follows.

Physical Quantity or Model Parameter	Unit
Capacity (compartment volume)	liter
First-order elimination rate	1/second
Km — Michaelis constant	milligram/liter
Vm — (Vmax) Maximum reaction-velocity (Michaelis-Menten kinetics)	milligram/second
Clearance	liter/second

Physical Quantity or Model Parameter	Unit
TkO (absorption duration)	second
ka (absorption rate)	1/second

Use the configuration settings options to turn unit conversion on or off. For more information see, "Model Simulation" on page 3-3 in the SimBiology documentation.

See "How Reaction Rates Are Evaluated" on page 1-22 in the SimBiology documentation for more information on dimensional analysis for reaction rates.

## **About Data Fitting in PKPD Models**

#### In this section ...

"Data Fitting Functionality" on page 4-29

"Prerequisites for Data Fitting" on page 4-30

"Prerequisites for Using Custom SimBiology Models in Data Fitting" on page 4-30

## **Data Fitting Functionality**

SimBiology lets you perform individual and population fitting on grouped data. This functionality uses Statistics Toolbox features (Version 7.3 or later).

- Individual fit Fit data separately for each individual using the nonlinear least squares method, estimate parameters, and calculate residuals and the estimated coefficient covariance matrix.
- Population fit Estimate the fixed effects and the random sources of variation on parameters, using nonlinear mixed-effects models.

You can use the following methods to estimate the fixed effects:

- LME Linear mixed-effects approximation
- RELME Restricted LME approximation
- F0 First-order estimate
- FOCE First-order conditional estimate

The following results are returned for population fitting:

- The maximized log-likelihood for the fitted model
- The estimated error variance for the fitted model
- The Akaike information criterion for the fitted model
- The Bayesian information criterion for the fitted model
- The standard errors for the estimates of the fixed effects
- The error degrees of freedom for the model

• The weighted residuals for the fitted model

In addition, you can generate diagnostic plots that show:

- The predicted time courses and observations for an individual or the population
- Observed versus predicted values
- Weighted residuals versus time, group, or predictions
- Distribution of the weighted residuals
- A box-plot for random effects or parameter estimates from individual fitting

#### **Prerequisites for Data Fitting**

Before you fit parameters, the SimBiology desktop or the MATLAB Workspace must contain the following:

- Data to use in the fitting (See "Importing Data Supported Files and Data Types" on page 4-8 for more information.)
- A model to fit (See "Creating Pharmacokinetic Models" on page 4-17 for more information.)

If you plan to use the command line, see the following for more information:

"Performing Data Fitting with PKPD Models" on page 4-34

#### Prerequisites for Using Custom SimBiology Models in Data Fitting

#### **Overview**

If you created a PK model using either the PKModelDesign object's construct method at the command line or the wizard in the SimBiology desktop, you can skip this section. This section provides information about working with a custom SimBiology model.

When using a custom model, you must provide information about whether dosing is applicable and define which components of the SimBiology model represent the observed response, the dose, and the estimated parameters. Use the PKModelMap object to define these settings as shown in "Defining Model Components for Observed Response, Dose, Dosing Type, and Estimated Parameters" on page 4-31.

#### Defining Model Components for Observed Response, Dose, Dosing Type, and Estimated Parameters

The PKModelMap object holds information about the dosing type and defines which components of the SimBiology model represent the observed response, the dose, and the parameters to be estimated.

If you are using a custom SimBiology model that you did not create using either the PKModelDesign object's construct method or the wizard, you must create a PKModelMap object to define these relationships.

Consider the following regression function, y = f(k, u), where y is the measured or observed response, given values of an input u, and parameter values k. In SimBiology, the model represents f, which is used to generate the regression function, if y, k, and u are identified in the model. You must, therefore, use the PKModelMap object to define which components of the model represent y, k, and u. If applicable, the PKModelMap object also needs information on the type of dosing or input being given to the model.

**1** Import an SBML model:

```
modelObj = sbmlimport('lotka');
```

2 Create a PKModelMap object:

PKModelMapObj = PKModelMap;

**3** Use the name of the model component to specify the corresponding property in the PKModelMap object.

Model Component Represents	PKModelMap Object Property	
Object being driven by an input	Dosed	
Measured response	Observed	
Parameters to be estimated	Estimated	

For example:

```
set(PKModelMapObj, 'Observed', 'unnamed.y1');
set(PKModelMapObj, 'Estimated', {'Reaction1.c1', 'Reaction2.c2'});
```

**Note** When specifying species names, qualify the name with the compartment name in the form compartmentName.speciesName (for example, nucleus.DNA). For names of parameters scoped at the reaction level, use reactionName.parameterName. For parameters scoped at the model level, you do not have to qualify the name.

**4** Use the DosingType property to specify the type of dosing, if applicable. The allowed types are '', 'Bolus', 'Infusion', 'FirstOrder', and 'ZeroOrder'.

For example:

```
set(PKModelMapObj, 'DosingType', 'Bolus');
```

**Note** When using custom models with DosingType set to zero-order, you must include a parameter that represents the duration of drug absorption. Set the ZeroOrderDurationParameter property of the PKModelMap object to the name of the duration parameter. For example, set(PKModelMapObj, 'ZeroOrderDurationParameter', 'Kdo');.

The previous example sets the observed response to a species y1, contained by a compartment (unnamed), and sets the parameters to be estimated to the parameters c1 and c2 that are scoped to the reactions, Reaction1 and Reaction2, respectively.

For information on	See
PKModelMap object properties and allowed values	PKModelMap object Dosed, DosingType, Estimated, and Observed, ZeroOrderDurationParameter
Allowed dosing types	"Dosing Types" on page 4-21

For information on	See
Parameter scoping	"When Reactions, Rules, and Events Specify Parameters" on page 1-17
Parameter fitting	"Performing Data Fitting with PKPD Models" on page 4-34

#### **Dosing Multiple Compartments in a Model**

1 Use the name of the model component to specify the Dosed property in the PKModelMap object.

For example, assume that a model contains two compartments named Central and Peripheral. Specify the species names in the dosed compartments. For example:

```
set(PKModelMapObj, 'Dosed, {'Central.Drug_Central', ...
'Peripheral.Drug_Peripheral'});
```

2 Use the DosingType property to specify the type of dosing if applicable. The allowed types are '', 'Bolus', 'Infusion', 'FirstOrder', and 'ZeroOrder'. When specifying dosing for multiple compartments, the order in the Dosed property is the order in which the dosing type is applied.

For example, if Central takes zero-order dosing and Peripheral takes a first-order dosing enter:

```
set(PKModelMapObj, 'DosingType', {'ZeroOrder', 'FirstOrder'});
```

**3** Because the model includes zero-order as a DosingType, you must include a parameter that represents the duration of drug absorption and is used when simulating the model with dosing information or during fitting. Set the ZeroOrderDurationParameter property of the PKModelMap object to the name of the duration parameter. For example,

```
set(PKModelMapObj, 'ZeroOrderDurationParameter', {'Kdo', ''})
```

Specify the parameters in the same order as the species in the Dosed property.

## Performing Data Fitting with PKPD Models

#### In this section ...

"Data Fitting Workflow" on page 4-34		
'Specifying and Classifying the Data to Fit" on page 4-35		
"Specifying Solver Type and Options for Fitting" on page 4-38		
"Setting Initial Estimates" on page 4-38		
"Specifying a Nonlinear, Mixed-Effects Model" on page 4-39		
"Specifying a Covariate Model" on page 4-41		
"Specifying the Covariance Pattern of Random Effects" on page 4-43		
"Specifying an Error Model" on page 4-45		
"Specifying Parameter Transformations" on page 4-45		
"Performing Population Fitting" on page 4-46		
"Performing Individual Fitting" on page 4-51		

### **Data Fitting Workflow**

The following steps show one of the workflows you can use at the command line to fit a PK model and estimate parameters:

- 1 Import data as shown in "Importing Data" on page 4-13.
- **2** Specify the structural model by creating a PK model as shown in "Creating Pharmacokinetic Model Using the Command Line" on page 4-19. Alternatively, if you have a SimBiology model that you want to use in fitting, see "Prerequisites for Using Custom SimBiology Models in Data Fitting" on page 4-30.
- **3** Classify the data set to use in fitting. See "Specifying and Classifying the Data to Fit" on page 4-35.
- **4** Specify the initial guesses for the parameters to be estimated, as shown in "Setting Initial Estimates" on page 4-38.
- **5** Perform individual or population fits:

- For individual fits:
  - (Optional) Specify an error model or weights. See "Specifying an Error Model" on page 4-45.
  - (Optional) Set tolerances.
  - (Optional) Specify maximum iterations.
  - (Optional) Specify to pool the data, which simultaneously fits data from multiple dose levels using the same model parameters for each dose.
- For population fits:
  - Specify the statistical model:

Specify the covariate model and the covariance matrix. See "Specifying a Covariate Model" on page 4-41 and "Specifying the Covariance Pattern of Random Effects" on page 4-43.

(Optional) Specify the error model. See "Specifying an Error Model" on page 4-45.

- (Optional) Set tolerances.
- (Optional) Specify maximum iterations.
- 6 Obtain and visualize results.

### Specifying and Classifying the Data to Fit

In order to use the imported data in fitting, you must identify required columns in the data set that was previously imported as shown in "Importing Data" on page 4-13.

Use the PKData object to specify the data set containing the observed data to use in fitting. The properties of the PKData object specify what each column in the data represents.

To create the PKData object:

1 Create the PKData object for the data set data.

```
pkDataObject = PKData(data);
```

PKData assigns the data set data to the read-only DataSet property.

**2** Use the column headers in the data set to specify the following properties for the column in the data set.

Column in Data Set Represents	PKData Object Property
Group identification labels	GroupLabel
Independent variable (For example, time)	IndependentVarLabel
Dependent variable (For example, measured response)	DependentVarLabel
Amount of dose given	DoseLabel
Rate of infusion (when applicable). Data must contain rate (amount/time) and not infusion time.	RateLabel
Covariates (For example, age, gender, weight)	CovariateLabels

For example, for the tobramycin data set [1]:

pkDataObject.GroupLabel	= 'ID';
pkDataObject.IndependentVarLabel	. = 'Time';
pkDataObject.DependentVarLabel	= 'Response';
pkDataObject.DoseLabel	= 'Dose';
pkDataObject.CovariateLabels	<pre>= {'WT', 'HT', 'AGE', 'SEX', 'CLCR'};</pre>

**Note** For the subset of data belonging to a single group (as defined by the column in your data set that represents group identification labels, which you map to the GroupLabel property), the software allows multiple observations made at the same time. If this is true for your data, be aware that:

- These data points are not averaged, but fitted individually.
- Different number of observations at different times cause some time points to be weighted more.

**Tip** If dosing applies to more than one compartment in the model, specify the DoseLabel property as follows:

```
pkDataObject.DoseLabel = {'Dose1', 'Dose2'};
```

Dose1 and Dose2 are names of columns containing dose information for compartments. A one-to-one relationship must exist between the number and order of elements in the DoseLabel property and the Dosed property of the corresponding PKModelMap object.

**Tip** If your model measures multiple responses, specify the DependentVarLabel property as follows:

```
pkDataObject.DependentVarLabel = {'Response1', 'Response2'};
```

Response1 and Response2 are names of columns containing response measurements. A one-to-one relationship must exist between the number and order of elements in the DependentVarLabel property and the Observed property of the corresponding PKModelMap object.

When you assign a column containing group identification labels to the GroupLabel property, PKData sets these read-only properties as follows:

• The GroupNames property is set to the unique names found in the group column.

• The GroupID property is set to an integer corresponding to the unique names found in the group column.

## **Specifying Solver Type and Options for Fitting**

If you specify a stochastic solver and options in the Configset object associated with your model, be aware that during fitting SimBiology temporarily changes:

- SolverType property to the default solver of ode15s
- SolverOptions property to the options last configured for a deterministic solver

### **Setting Initial Estimates**

**Caution** If your model includes active variants that specify alternate values for the parameters to estimate, the variants are ignored for those parameters during fitting.

To set the initial estimates (or initial guesses) for the parameters with fixed effects to estimate, first identify the sequence of the parameters in the model by querying the PKModelMap object. Next, construct a vector, beta0, containing the initial conditions. For information about PKModelMap objects, see step 4 in "Creating Pharmacokinetic Model Using the Command Line" on page 4-19.

1 Query the Estimated property of the PKModelMap object:

```
PKModelMapObj.Estimated
```

MATLAB returns the sequence of the parameters to be estimated. For example:

```
ans =
```

```
'Central'
'Cl_Central'
```

2 Set the initial estimates for the parameters. For example:

beta0 = [10.0, 1.0];

For information on	See
The parameters added to the model	<ul><li>"Dosing Types" on page 4-21</li><li>"Elimination Types" on page 4-24</li></ul>
Default units for the above parameters	"Unit Conversion for Imported Data" on page 4-27

# Specifying a Nonlinear, Mixed-Effects Model

Suppose data for a nonlinear regression model falls into one of m distinct groups i = 1, ..., m. (Specifically, suppose that the groups are not nested.) To specify a general, nonlinear, mixed-effects (NLME) model for this data:

- 1 Define group-specific model parameters  $\varphi_i$  as linear combinations of fixed effects  $\beta$  and random effects  $b_i$ .
- **2** Define response values  $y_i$  as a nonlinear function *f* of the parameters and group-specific covariate variables  $X_i$ .

The model is:

$$\begin{split} \varphi_i &= A_i \theta + B_i \eta_i, \text{where } \eta_i \square N(0, \Psi) \\ y_i &= f(\varphi_i, X_i) + \varepsilon_i \\ \text{Alternatively, } \log y_i &= \log f(\varphi_i, X_i) + \varepsilon_i \end{split}$$

This formulation of the nonlinear, mixed-effects model uses the following notation:

- $\varphi_i$  A vector of group-specific model parameters
- $\theta$  A vector of fixed effects, modeling population parameters
- $\eta_i$  A vector of multivariate, normally distributed, group-specific, random effects

- $A_i$  A group-specific design matrix for combining fixed effects
- $B_i$  A group-specific design matrix for combining random effects
- $X_i$  A data matrix of group-specific covariate values
- $y_i$  A data vector of group-specific response values
- f A general, real-valued function of  $\varphi_i$  and  $X_i$
- For sbionlmefit, you can specify different error models as shown in "Specifying an Error Model" on page 4-45.
  - For sbionlmefitsa, you can specify different error models as shown in "Specifying an Error Model" on page 4-45.
- $\Psi$  A covariance matrix for the random effects
- $\sigma^2$  The error variance, assumed to be constant across observations

For example, consider a one-compartment model with first-order dosing and linear clearance. The group-specific parameters ( $\varphi$ ) in the model are clearance (*Cl*), compartment volume (*V*), and absorption rate constant ( $k_a$ ). From the model:

(Cl)		(1	0	0)	$\left( \theta_{Cl} \right)$		(1	0	0)	$(\eta_{Cl})$
	=	0	1	0	$\theta_{v}$	+	0	1	0	$\eta_v$
(ka)		0	0	1)	$\left( \theta_{ka} \right)$		0	0	1)	$\begin{pmatrix} \eta_{Cl} \\ \eta_{V} \\ \eta_{ka} \end{pmatrix}$

In SimBiology,  $B_i$  is an identity matrix. That is, sbionlmefit does not support the specification of a different random-effects design matrix. You can alter the design matrices, as necessary, to introduce weighting of individual effects.

The Statistics Toolbox function nlmefit fits the general, nonlinear, mixed-effects model to data, estimating the fixed and random effects. The function also estimates the covariance matrix  $\Psi$  for the random effects. Additional diagnostic outputs allow you to assess trade-offs between the number of model parameters and the goodness of fit. See "Mixed-Effects Models" in the Statistics Toolbox documentation for more information.

# Specifying a Covariate Model

Construct a CovariateModel object to define the relationship between parameters and covariates. After constructing the object, modify the FixedEffectValues property of the object before using the object as an input argument to sbionlmefit or sbionlmefitsa, to estimate nonlinear mixed effects.

If the NLME model assumes a group-dependent covariate such as weight (w), the model becomes:

$ \begin{pmatrix} Cl \\ V \\ ka \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & w_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \theta_{Cl} \\ \theta_{V} \\ \theta_{ka} \\ \theta_{Cl/w} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{Cl} \\ \eta_{V} \\ \eta_{ka} \end{pmatrix} $	$\begin{pmatrix} Cl\\V\\ka \end{pmatrix} = \begin{pmatrix} 1\\0\\0 \end{pmatrix}$	0 1 0	0 0 1	$egin{array}{c} w_i \ 0 \ 0 \end{array}  ight)$	$ \begin{pmatrix} \theta_{cl} \\ \theta_{v} \\ \theta_{ka} \\ \theta_{rad} \end{pmatrix} $	$+ \begin{pmatrix} 1\\0\\0 \end{pmatrix}$	0 1 0	$ \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \begin{pmatrix} \eta_{cl} \\ \eta_{v} \\ \eta_{ka} \end{pmatrix} $
--	---	-------------	-------------	---	--	---	-------------	---

Thus, the parameter for clearance (Cl) for an individual is  $Cl_i = \theta_{Cl} + \theta_{Cl/w} * w_i + \eta_{Cl}$ .

Use the following procedure to specify a covariate model. If you are using the tobramycin data set, make sure you first complete the following procedures:

- "Importing Data from NONMEM Formatted Files" on page 4-14
- "Creating Pharmacokinetic Model Using the Command Line" on page 4-19
- "Specifying and Classifying the Data to Fit" on page 4-35
- "Setting Initial Estimates" on page 4-38
- 1 Use the CovariateModel constructor function to construct an empty CovariateModel object:

covModel = CovariateModel;

2 Set the Expression property of the object to define the relationship between parameters and covariates in the CovariateModel object, where Cl, v, and ka are parameters, weight is a covariate, theta1, theta2, theta3, and theta4 are fixed effects, and eta1, eta2, and eta3 are random effects.

```
covModel.Expression = {'Cl = exp(theta1 + theta4*weight + eta1)',...
'v = exp(theta2 + eta2)',...
```

'ka = exp(theta3 + eta3)'};

**3** Display a list of the descriptions of the fixed effects (theta1 and theta2) in the CovariateModel object:

```
disp('Fixed Effects Descriptions:');
disp(covModel.FixedEffectDescription)
```

Your output appears as follows, where each string describes the role of a fixed effect in the expression equation:

```
Fixed Effects Descriptions:

'Cl'

'v'

'ka'

'Cl/weight'
```

**4** Use the constructDefaultFixedEffectValues method of the CovariateModel object to create a structure containing the initial estimates for the fixed effects in the object. The initial estimates in this structure are set to a default of zero:

initialEstimates = covModel.constructDefaultFixedEffectValues

Your output appears as:

initialEstimates =

theta1: 0
theta2: 0
theta3: 0
theta4: 0

**5** Edit the initialEstimates structure to set the initial estimates of the fixed effects:

initialEstimates.theta1 = 1.408; initialEstimates.theta2 = 0.061; initialEstimates.theta3 = 0.31; **Tip** Typically, these initial estimates are values you determine from a previous fit of the data.

**6** Use the modified initialEstimates structure to update the FixedEffectValues property of the CovariateModel object:

```
covModel.FixedEffectValues = initialEstimates;
```

Now covModel, the CovariateModel object, is ready to submit as an input argument to sbionlmefit or sbionlmefitsa.

# **Specifying the Covariance Pattern of Random Effects**

By default, the function you use to perform population fits (nlmefit or nlmefitsa) assumes a diagonal covariance matrix (no covariance among the random effects). To specify a different covariance pattern of random effects, use the 'CovPattern' option. In the previous example, assuming that each of the parameters has random effects and that *Cl* and *V* exhibit covariance, the covariance pattern of random effects would be a logical array:

```
 \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
```

1 Create an options struct with the specified covariance pattern:

options.CovPattern = [1, 1, 0; 1, 1, 0; 0, 0, 1];

2 Specify the arguments for sbionlmefit or sbionlmefitsa:

```
[results, simdataI, simdataP] = sbionlmefit(modelObj,...
PKModelMapObj, pkDataObject, betaO, options)
```

If you are using the tobramycin data set [1], do the following:

1 Create an options struct with the specified covariance pattern:

options.CovPattern = [1, 0; 0, 1];

```
2 Specify the arguments for sbionlmefit:
  [results, simdataI, simdataP] = sbionlmefit(modelObj,...
       PKModelMapObj, pkDataObject, betaO, options)
  results =
   NLMEResults handle
   Properties:
                      FixedEffects: [2x3 dataset]
                     RandomEffects: [97x2 dataset]
      IndividualParameterEstimates: [97x2 dataset]
      PopulationParameterEstimates: [97x2 dataset]
     RandomEffectCovarianceMatrix: [2x2 dataset]
           EstimatedParameterNames: {2x1 cell}
                    CovariateNames: {'WT' 'HT'
                                                   'AGE '
                                                          'SEX'
                                                                 'CLCR'}
                FixedEffectsStruct: [1x1 struct]
                              stats: [1x1 struct]
  results.FixedEffects
 ans =
     Description
                           Estimate
                                       StandardError
      'Central'
                           3.0478
                                       0.064369
```

For more information, see nlmefit or nlmefitsa in the Statistics Toolbox documentation.

0.061095

1.3054

'Cl Central'

Fitting the model and estimating the covariance matrix  $\Psi$  often leads to further refinements. A relatively small estimate for the variance of a random effect suggests that it can be removed from the model. Similarly, relatively small estimates for covariances among certain random effects suggest that a full covariance matrix is unnecessary. Since random effects are unobserved,  $\Psi$  must be estimated indirectly. Specifying a diagonal or block-diagonal covariance pattern for  $\Psi$  can improve convergence and efficiency of the fitting algorithm.

# **Specifying an Error Model**

You can specify any of the following error models when using the sbionlinfit, sbionlmefit, or sbionlmefits function. Each model defines the error using a standard normal (Gaussian) variable e, the function value f, and one or two parameters, a and b. The default error model is 'constant'.

- 'constant':  $y = f + a^*e$  (default)
- 'proportional': y = f + b\*abs(f)\*e
- 'combined': *y* = *f* + (*a*+*b*\*abs(*f*))\**e*
- 'exponential':  $y = f^* \exp(a^* e)$ , or equivalently  $\log(y) = \log(f) + a^* e$

To define an error model:

1 Create an optionStruct input argument and set the ErrorModel field to specify one of the above error models. For example:

```
optionStruct.ErrorModel = 'proportional';
```

2 Specify the optionStruct input argument for sbionlinfit, sbionlmefit, or sbionlmefitsa, as shown in "Performing Individual Fitting" on page 4-51 or "Performing Population Fitting" on page 4-46.

See also or nlinfit, nlmefit, or nlmefitsa in the Statistics Toolbox documentation.

### **Specifying Parameter Transformations**

To specify parameter transformations, use the ParamTransform option in sbionlinfit, sbionlmefit and sbionlmefitsa. The ParamTransform option lets you specify either no transformation, or the log, probit, or logit transformation.

**Note** Do not use the ParamTransform option to specify parameter transformations when providing a CovariateModel object to a fitting function. The CovariateModel object provides the parameter transformation.

The underlying algorithm in nlmefit assumes that parameters follow a normal distribution. This assumption may not hold for biological parameters that are constrained to be positive, such as volume and clearance. You may specify a transformation function for the estimated parameters, so that the transformed parameters follow a normal distribution.

By default, the SimBiology fitting functions choose a log transform for all estimated parameters. Parameters that are constrained between the values 0 and 1, like absorption fraction, can be transformed by the probit or logit transformations described below.

The probit function is the inverse cumulative distribution function (CDF) associated with the standard normal distribution. To apply the probit transform to a variable x in MATLAB, use the Statistics Toolbox function norminv: t = norminv(x). To untransform a variable t, use the function normcdf: x = normcdf(t).

The logit function is the inverse of the sigmoid function. To apply the logit transform to a variable x in MATLAB, use the following expression: t = log(x) - log(1-x). To untransform the variable t, use x = 1/(1+exp(-t)).

1 For the ParamTransform option, specify a vector of values equal to the number of parameters to be estimated. The values must be one of the integer codes listed in nlmefitsa or nlmefit specifying the transformation for the corresponding value of the parameters to be estimated. For example

options.ParamTransform = [0 1 2];

See nlmefit and nlmefitsa for more information.

**2** Specify the arguments for sbionlmefit or sbionlmefitsa, as shown in "Performing Population Fitting" on page 4-46.

For individual fitting, see "Performing Individual Fitting" on page 4-51.

### **Performing Population Fitting**

The sbionlmefit and sbionlmefitsa functions let you specify a SimBiology model that you want to use in fitting. These functions use the nlmefit and nlmefitsa functions from the Statistics Toolbox to fit data with both fixed and random sources of variation using nonlinear mixed-effects and return

the estimates. Both nlmefitand nlmefitsa fit the model by maximizing an approximation to the marginal likelihood with random effects integrated out assuming the following:

- Random effects are multivariate, normally distributed, and independent between groups.
- Observation errors are independent, identically normally distributed, and independent of random effects.
- 1 (Optional) Set the tolerance or maximum iteration options. Use an options structure that is an input argument for sbionlmefit or sbionlmefitsa:

```
optionStruct.Options.TolX = 1.0E-4;
optionStruct.Options.TolFun = 1.0E-4;
optionStruct.Options.MaxIter = 200;
```

2 Specify the model object, the PKModelMap object, the PKData object, the PKCovariateModel object, a vector containing the initial estimates for the fixed effects, and the options:

```
[results, simdataI, simdataP] = sbionlmefit(modelObj,...
PKModelMapObj, pkDataObject, CovariateModelObject, betaO, optionStruct);
```

**Note** If your population fit uses multiple doses, make sure each element in the Dosed property of the PKModelMap object is unique.

**Note** In your PKData object, for each subset of data belonging to a single group (as defined in the data column specified by the GroupLabel property), the software allows multiple observations made at the same time. If this is true for your data, be aware that:

- These data points are not averaged, but fitted individually.
- Different number of observations at different times cause some time points to be weighted more.

sbionlmefit and sbionlmefitsa return the following:

- *results*, an object containing estimated values and other statistics. For more information, see the sbionlmefit and sbionlmefitsa reference pages.
- *simdataI*, a SimData object containing the data from simulating the model using the estimated parameter values for individuals, which includes both the fixed and random effects.
- *simdataP*, SimData object containing the data from simulating the model using the estimated parameter values for the population, which includes only the fixed effects.
- **3** Plot the data from the data set. For example, in the imported data set used for fitting, ds, ID, Time, and Response are the column headers for the columns containing group IDs, time, and the response variable, respectively.

```
p = sbiotrellis(ds, 'ID', 'Time', 'Response')
```

**Note** If your data set has multiple responses, with column headers Response1 and Response2 containing the response variables, you plot the data as follows:

```
Response = {'Response1', 'Response2'}
p = sbiotrellis(ds, 'ID', 'Time', Response)
```

**4** Use the plot method on the trellis plot object p, returned by sbiotrellis to overlay data, using default values for the second and third input arguments.

```
p.plot(simdataP, [], '', PKModelMapObj.Observed);
```

For a description of the results, see sbionlmefit in the SimBiology documentation.

For more information, see the following topics in the Statistics Toolbox documentation:

• "Nonlinear Regression"

- "Mixed-Effects Models"
- nlmefit

### Simultaneously Fitting Data from Multiple Dose Levels

When performing population fitting using nonlinear regression, you can simultaneously fit data from multiple dose levels by either:

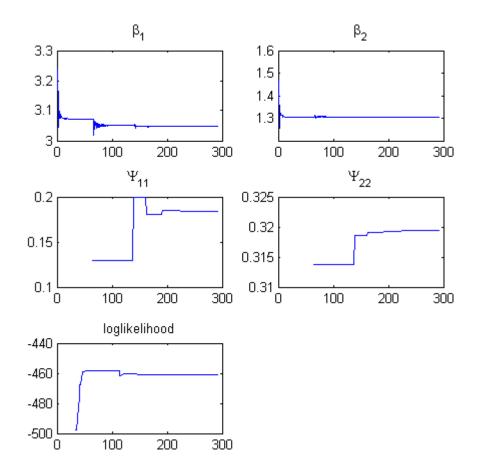
- Using sbionlmefit with a CovariateModel object input argument and omitting the random effect (eta) from the expressions in the CovariateModel object.
- Using sbionlmefit with an InitEstimates input argument and setting the REParamsSelect field or name-value pair input argument to a 1-by-*n* logical vector, with all entries set to false, where *n* equals the number of fixed effects.

### **Obtaining the Status of Fitting**

The sbiofitstatusplot function dynamically plots the progress of the fitting task. During the task, the function plots the fixed effects ( $\beta$ ), the estimates for the diagonal elements of the covariance matrix for the random effects ( $\Psi$ ), and the log-likelihood. This functionality is useful for large and complex models when you expect the time to return the results to be longer than a few minutes. Use the options structure that is an argument for the sbionlmefit function:

```
% Create options structure with 'OutputFcn'.
options.Options.OutputFcn = @sbiofitstatusplot;
% Pass options structure with OutputFcn to sbionlmefit function.
results = sbionlmefit(..., options);
```

The following figure shows the type of plots obtained.



Tips for interpreting status plots:

- The fitting function tries to maximize the log-likelihood. When the plot begins to display a flat line, this might indicate that maximization is complete. Try setting the maximum iterations to a lower number to reduce the number of iterations you need and improve performance. For information on how to set iteration options, see "Performing Population Fitting" on page 4-46.
- Plots for the fixed effects (β) and the estimates for the diagonal elements of the covariance matrix for the random effects (Ψ), should show convergence.

If you see oscillations, or jumps without accompanying improvements in the log-likelihood, the model may be over-parameterized. Try the following:

- Reduce the number of fixed effects.
- Reduce the number of random effects.
- Simplify the covariance matrix pattern of random effects.

See also sbiofitstatusplot in the SimBiology documentation.

## **Performing Individual Fitting**

The sbionlinfit function lets you specify a SimBiology model to fit using the nlinfit function (individual fit). The nlinfit function uses nonlinear least squares and returns parameter estimates, residuals, and the estimated coefficient covariance matrix.

1 (Optional) Specify an error model, set the tolerance, set the maximum iteration, or set the data pooling option, which lets you simultaneously fit data from multiple dose levels using the same model parameters for each dose. Use an options structure that is an input argument for sbionlinfit:

```
optionStruct.ErrorModel = 'proportional';
optionStruct.TolX = 1.0E-8;
optionStruct.TolFun = 1.0E-8;
optionStruct.MaxIter = 100;
optionStruct.Pooled = true;
```

2 Specify the model object, the PKModelMap object, the PKData object, a vector containing the initial estimates for the fixed effects, and the options:

```
[results, simdataI] = sbionlinfit(modelobj,...
PKModelMapObj, PKDataObj, betaO, optionStruct);
```

**Note** If your individual fit uses multiple doses, ensure each element in the Dosed property of the PKModelMap object is unique.

**Note** In your PKData object, for each subset of data belonging to a single group (as defined in the data column specified by the GroupLabel property), the software allows multiple observations made at the same time. If this is true for your data, be aware that:

- These data points are not averaged, but fitted individually.
- Different number of observations at different times cause some time points to be weighted more.

sbionlinfit returns the following:

- A *results* array of objects, with each object containing the following for one group:
  - ParameterEstimates A dataset array containing fitted coefficients and their standard errors.
  - CovarianceMatrix Estimated covariance matrix for the fitted coefficients.
  - beta Vector of scalars specifying the fitted coefficients in transformed space.
  - R Residuals.
  - J Jacobian of modelObject.
  - COVB Estimated covariance matrix for the transformed coefficients.
  - mse Scalar specifying the estimate of the error of the variance term.
  - errorparam Estimated parameters of the error model or an empty array if you specified weights using the 'Weights' name-value pair argument.
- *simdataI*, a SimData object containing the data from simulating the model using the estimated parameter values, for individuals.
- 3 Plot the data from the data set. For example, in the imported data set (ds), ID, Time and Response are the column headers for the columns containing group IDs, time, and the response variable respectively.

p = sbiotrellis(ds, 'ID', 'Time', 'Response')

**Note** If your data set has multiple responses, with column headers Response1 and Response2 containing the response variables, then plot the data as follows:

```
Response = {'Response1', 'Response2'}
p = sbiotrellis(ds, 'ID', 'Time', Response)
```

**4** Use the plot method on the trellis plot object p, returned by sbiotrellis to overlay data, using default values for the second and third input arguments.

```
p.plot(simdataI, [], '', PKModelMapObj.Observed);
```

For more information, see "Nonlinear Regression" and nlinfit in the Statistics Toolbox documentation.

# **PK/PD Modeling and Simulation to Guide Dosing Strategy** for Antibiotics

This example shows how to perform a Monte Carlo simulation of a pharmacokinetic/pharmacodynamic (PK/PD) model for an antibacterial agent. This example is adapted from Katsube et al. [1] This example also shows how to use the SimBiology exported model to perform parameter scans in parallel.

This example requires Statistics Toolbox<sup>™</sup>. The performance can be improved if you have the Parallel Computing Toolbox<sup>™</sup> software.

#### Background

Katsube et al. [1] used a PK/PD modeling and simulation approach to determine the most effective dosage regimens for doripenem, a carbapenem antibiotic. The objectives of their study were:

- Develop a PK/PD model to describe the antibacterial effect of doripenem against several Pseudomonas aeruginosa strains
- Use Monte Carlo simulations to compare the efficacy of four common antibiotic dosage regimes, and to determine the most effective dosing strategy
- Investigate the effect of renal function on the antibacterial efficacy of the treatments

In this example, we will implement the antibacterial PK/PD model developed by Katsube et al. [1] in SimBiology, and replicate the results of the Monte Carlo simulation described in their work.

#### References

[1] T. Katsube, Y. Yano, T. Wajima, Y. Yamano and M. Takano. Pharmacokinetic/pharmacodynamic modeling and simulation to determine effective dosage regimens for doripenem. *Journal of Pharmaceutical Sciences* (2010) 99(5), 2483-91.

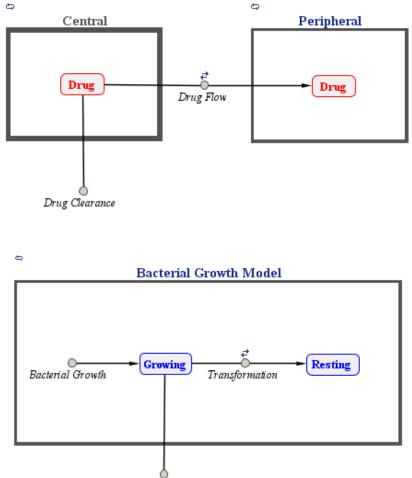
#### **PK/PD Model**

Katsube et al. assumed a two-compartment infusion model with linear elimination from the central compartment to describe the pharmacokinetics of the doripenem. For the bacterial growth model, they assumed that the total bacterial population is comprised of drug-susceptible growing cells and drug-insensitive resting cells. The antibacterial effect of the drug was included in the killing rate of the bacteria via a simple Emax type model:

$$KillingRate = rac{Kmax * [Drug] * [Growing]}{KC50 + [Drug]}$$

where [Drug] is the concentration (ug/ml) of the drug in the central compartment, and [Growing] is the count of the growing bacterial population in CFU/ml (CFU = Colony Forming Units). Kmax is the maximal killing rate constant (1/hour) and KC50 is the Michaelis-Menten rate constant (ug/ml).

A graphical view of the SimBiology implementation of the model is shown below.



Drug-Depen...Bacterial Death

```
% Load model
sbioloadproject('AntibacterialPKPD.sbproj', 'm1');
```

#### **Dosage Regimens**

Katsube et al. simulated the model using four common antibiotic dosage strategies.

- 250 mg two times a day (b.i.d.)
- 250 mg three times a day (t.i.d.)
- 500 mg two times a day (b.i.d.)
- 500 mg three times a day (t.i.d.)

Infusion dosing was used in all four dosages regimens, and infusion time was set to 30 minutes. In SimBiology, these dosage regimens have been implemented as dose objects.

#### **Description of the Virtual Population**

A virtual population of individuals was generated based on the distribution of demographic variables and PK/PD parameters. The type of distribution and the values of the distribution parameters were based on data from earlier clinical trials of doripenem conducted in Japan.

Note: In [1], 5,000 virtual patients were simulated in each dosage group. In this example, we will use 1,000 patients in each group. To simulate a different population size, change the value of nPatients below.

% Setup			
nPatients	= 1000	; % Number of patients per dosage group	С
nDoseGrps	= 4	; % Number of tested dosage regimens	

#### **Distribution of Demographic Variables:**

Weight (Wt) and age (Age) were sampled from a normal distribution with a mean of 51.6 kg and 71.8 years, respectively, and a standard deviation of 11.8 kg and 11.9 years, respectively. 26% of the population was assumed to be female. Serum creatinine levels (Scr) were sampled from a lognormal

distribution with a typical value of 0.82 mg/dL, and coefficient of variation (CV) of 32%. The creatinine clearance rates (CrCL) were calculated using the Cockcroft-Gault equation.

% Note: The inputs to the lognrnd function are the mean (mu) and standard % deviation (sigma) of the associated normal distribution. Here and % throughout the example, mu and sigma were calculated from the reported % typical value and coefficient of the lognormal distriution. See the % lognstat documentation for more information.

```
% Patient demographics
Wt
    = normrnd(51.6
                        , 11.8 , nPatients , nDoseGrps )
                                                           ; % units: kg
                       , 11.9 , nPatients , nDoseGrps ) ; % units: year
Age = normrnd(71.8
Scr = lognrnd(-0.2485 ,0.3197 , nPatients , nDoseGrps ) ; % units: ml/m
% Gender ratio
id
        = 1:nPatients*nDoseGrps
idFemale = randsample(id , round(0.26*nDoseGrps*nPatients)) ; % 26% Female
% Creatinine Clearance (using Cockcroft-Gault equation)
CrCL
               = (140 - Age).*Wt./(Scr*72)
                                                           ; % units: ml/m
CrCL(idFemale) = CrCL(idFemale)*0.85
                                                           ; % multiply by
```

#### **Distribution of Pharmacokinetic (PK) parameters:**

PK parameters, Central, k12 and k21, were sampled from a lognormal distribution with typical values of 7.64 liters, 1.59 1/hour and 2.26 1/hour, respectively, and a 20% coefficient of variation (CV). Central is the distribution volume of the central compartment, and k12 and k21 are transfer rate constants between the Central and the Peripheral compartments. The drug clearance rate, CL, was assumed to depend linearly on the creatinine clearance rate via the following equation:

 $CL = 1.07 * CrCL + 45.6 + \varepsilon$ 

where  $\varepsilon$  is the additive residual error sampled from a normal distribution with a mean of 0 ml/minute and standard deviation of 22 ml/minute.

```
% PK parameters
Central = lognrnd(2.01  , 0.2 , nPatients, nDoseGrps)        ; % units: l
```

k12	= lognrnd(0.4441 , 0.2 , nPatients, nDoseGrps)	;	% units: 1
k21	= lognrnd(0.7958 , 0.2 , nPatients, nDoseGrps)	;	% units: 1
CL	= 1.07*CrCL + 45.6 + normrnd(0,22,nPatients,nDoseGrps)	;	% units: m

#### Distribution of Pharmacodynamic (PD) parameters:

Growing-to-resting transformation rate constants, k1 and k2, were sampled from a lognormal distribution with typical value of 5.59e-5 and 0.0297 1/hour, respectively, each with a CV of 20%. Kmax was sampled from a lognormal distribution with a typical value of 3.5 1/hour and 15.9% CV. Katsube et al. assumed that values k1, k2 and Kmax were independent of the bacterial strain being treated. The value of Beta, the net growth rate constant, was fixed at 1.5 1/hour.

Based on experiments with several strains, the authors concluded that the value of KC50 was linearly dependent on the minimum inhibition concentration (MIC) of bacterial strain via the following equation.

 $ln(KC50) = -1.91 + 0.898 * ln(MIC) + \varepsilon$ 

where  $\varepsilon$  is the additive residual error sampled from a normal distribution with a mean of 0 and standard deviation of 1.06 ug/ml. In the simulation, the MIC values were sampled from a discrete distribution, and the KC50 value was calculated for the selected MIC using the above equation.

```
% Discrete distribution of MIC values based on 71 P. aeruginosa strains
micValue = [0.0625, 0.125, 0.25, 0.5 , 1 , 2 , 4 , 8 , 16 , 32 ];
               5 , 8 , 9 , 14 , 7 , 8 , 9 , 5 , 2 , 4 ];
micFreq
         = [
k1
    = lognrnd(-9.8116, 0.2 , nPatients, nDoseGrps)
                                                           ; % units: 1/hc
k2
    = lognrnd(-3.5362, 0.2 , nPatients, nDoseGrps)
                                                           ; % units: 1/hc
Kmax = lognrnd( 1.2332, 0.159, nPatients, nDoseGrps)
                                                           ; % units: 1/hc
% Sample MIC values from a discrete distribution using randsample
MIC = nan(nPatients, nDoseGrps) ; % preallocate
for iDoseGrp = 1:nDoseGrps
    MIC(:, iDoseGrp) = randsample(micValue , nPatients, true , micFreq);
end
KC50 = exp(-1.91 + 0.898*log(MIC) + 1.06*randn(nPatients , nDoseGrps)) ; %
```

#### Simulation Setup & Design

You will convert the model to a SimBiology exported model, which facilitates performing parameter scans in parallel. When you export the model, you choose which species, parameters, or compartments can be varied. In this example, you will vary 8 parameters, Central, k12, k21, CL, k1, k2, Kmax, and KC50.

```
% Select the parameters you want to vary.
paramNames = {'Central', 'k12', 'k21', 'CL', 'k1', 'k2', 'Kmax', 'KC50'};
for iParam = 1:length(paramNames)
    parameters(iParam) = sbioselect(m1, 'Name', paramNames{iParam});
end
```

% Created the exported model, using the selected parameters and doses exportedModel = export(m1, parameters, doseRegimens);

```
% Accelerate the model
accelerate(exportedModel);
```

For all dosage scenarios, the model was simulated until t = 2 weeks from the time of the first dose. Total bacterial count, CFU, was sampled every 24 hours (once a day) for the entire duration of the dosage regimen.

```
tObs = 0:24:336 ; % hour
nTPoints = length(tObs) ; % Number of sampling points
% Specify that the simulation should report these output times
exportedModel.SimulationOptions.OutputTimes = tObs;
```

#### **Start the Worker Pool**

If you have the Parallel Computing Toolbox software, you can use local workers to distribute each simulation to a different worker. If you also have the MATLAB Distributed Computing Server<sup>™</sup>, you can run your simulations on a cluster.

You can use workers from the default parallel profile with this command

matlabpool open

#### Monte Carlo Simulation of Patients with Severe Infection

The antibacterial efficacy of a drug can be measured using different PK/PD indices. Katsube et al. set the criterion for bacterial elimination at log10(CFU) < 0, where CFU is the total bacterial count. The efficacy of each dose regimen was measured as the fraction of the population that achieved the success criteria in the dosage group. This efficacy metric, Pr{log10(CFU) < 0}, was tracked as a function of time for each dosage group.

In their simulation studies, the authors investigated the efficacy of the dosage regimens on two classes of patients:

- Moderate infection (Initial bacterial count = 1e4 CFU/ml)
- Severe infection (Initial bacterial count = 1e7 CFU/ml)

In this example, we will replicate the results for the severe infection case only. Note that you can easily simulate the other scenario, patients with moderate infection, by changing the initial amount of bacterial count in the model to 1e4 CFU/ml.

```
% Preallocate
log10CFU = cell(1,nDoseGrps) ;
for iDoseGrp = 1:nDoseGrps
% Select the exported doess
currentDose = getdose(exportedModel, doseNames{iDoseGrp});
cfu = nan(nTPoints , nPatients) ; % preallocate
disp(['Simulating group ', num2str(iDoseGrp), ' ... '])
parfor iPatient = 1:nPatients
% Use parameter values for current patient
% Define the parameters in the same order used when exporting
parameterValues = [
    Central(iPatient , iDoseGrp)
```

```
k12(iPatient, iDoseGrp)
           k21(iPatient, iDoseGrp)
           CL(iPatient , iDoseGrp)
           k1(iPatient , iDoseGrp)
           k2(iPatient , iDoseGrp)
           Kmax(iPatient, iDoseGrp)
           KC50(iPatient, iDoseGrp)
           ];
        % Simulate
        simData = simulate(exportedModel, parameterValues, currentDose) ;
        % Extract bacterial count data for Growing and Resting population
        [~, bactCount] = selectbyname(simData, {'Growing', 'Resting'});
        % Sum of growing and resting bacterial
        cfu(:, iPatient) = sum(bactCount, 2) ;
    end
    % Calculate log10(CFU)
    log10CFU{iDoseGrp} = log10(cfu) ;
end
% Save results
log10CFU_250bid = log10CFU{1} ;
log10CFU_250tid = log10CFU{2} ;
log10CFU 500bid = log10CFU{3};
log10CFU = log10CFU{4};
Simulating group 1 ...
Simulating group 2 ...
Simulating group 3 ...
Simulating group 4 ...
```

#### **Clean Up the Worker Pool**

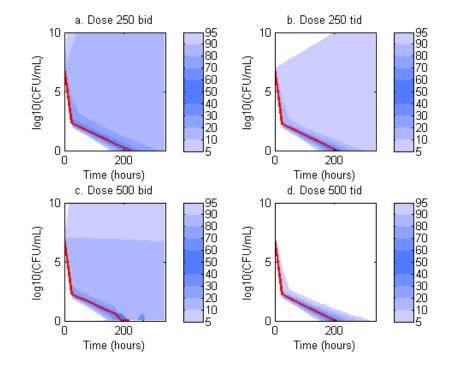
If you called matlabpool to start some workers, be sure to close them.

matlabpool close

#### **Time Course Profiles of Bacterial Counts**

We plot the median (in red) and percentile (shaded) profiles of the log10(CFU) levels for all four dosage regimens. Observe that in all four groups, the median time course profile shows that bacterial eradication is complete before the end of the treatment period (336 hours). However, it is evident from the higher percentile profiles that the treatments are not successful for all patients. The 95th and 90th percentile profiles also indicate that dosing a lower amount with a higher frequency (250 tid) is more effective than less frequent dosing with higher amount (500 bid).

```
hax1(1) = subplot(2,2,1) ; plotCFUCount(tObs, log10CFU_250bid, 'a. Dose 250
hax1(2) = subplot(2,2,2) ; plotCFUCount(tObs, log10CFU_250tid, 'b. Dose 250
hax1(3) = subplot(2,2,3) ; plotCFUCount(tObs, log10CFU_500bid, 'c. Dose 500
hax1(4) = subplot(2,2,4) ; plotCFUCount(tObs, log10CFU_500tid, 'd. Dose 500
% Link subplot axes
linkaxes(hax1)
```



Effect of Renal Function on Antibacterial Activity

Finally, the authors compared the efficacy profiles of the dosages regimens as a function of the renal function. They classified the patients into four renal function groups based on the creatinine clearance rates (CrCL):

- Creatinine Clearance Group 1: CrCL < 30
- Creatinine Clearance Group 2: 30 <= CrCL < 50
- Creatinine Clearance Group 3: 50 <= CrCL < 70
- Creatinine Clearance Group 4: CrCL >= 70

The next figure shows the effect of renal function (creatinine clearance rate) on the antibacterial efficacy of the four dosage regimens. Observe that in the normal renal function group (CrCL >= 70), the efficacy profiles of the four treatment strategies are significantly different from each other. In this case,

the 500 mg t.i.d. dose is much more effective than the other regimens. In contrast, simulations involving patients with renal dysfunction (CrCL < 30 and  $30 \le$  CrCL < 50), we don't see much difference between the treatment groups. This indicates that for patients with a renal dysfunction, a less intense or less frequent dosing strategy would work almost as well as a dosing strategy with higher frequency or dosing amount.

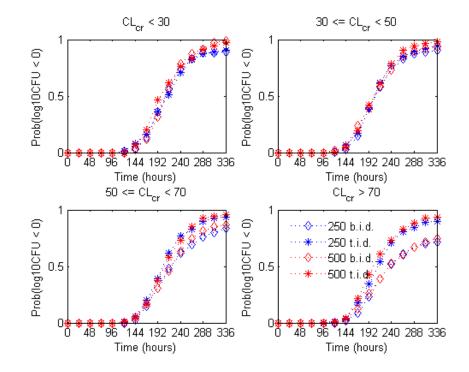
```
% Preallocate
idCrCLGrp = false(nPatients, nDoseGrps);
% Line Style
ls = {'bd:', 'b*:', 'rd:', 'r*:'};
titleStr = {'CL c r < 30'
                                    , ...
            '30 <= CL c_r < 50'
                                     , ...
            '50 <= CL c r < 70'
                                    , ...
            'CL c r > 70'
                                     };
f = figure;
set(f, 'Color', 'w')
for iCrCLGrp = 1:4 % Creatinine Clearance Groups
    hax2(iCrCLGrp) = subplot(2,2, iCrCLGrp) ;
    title( titleStr{iCrCLGrp}
                              );
    ylabel('Prob(log10CFU < 0)' );</pre>
    xlabel('Time (hours)'
                           );
end
% Set axes properties
set(hax2,
            'XTick'
                           , 0:48:336
                         , 0:48:336
            'XTickLabel'
                                            , ...
            'Ylim'
                           , [0 1]
                                            . . . .
            'Xlim'
                           , [0 336]
                                            , ...
            'NextPlot'
                           , 'add'
                                            , ...
            'Box'
                           , 'on'
                                           );
% Plot results by renal function group:
```

```
for iDoseGrp = 1:nDoseGrps
```

#### end

```
legend(hax2(4), {'250 b.i.d.', '250 t.i.d.', '500 b.i.d.', '500 t.i.d.'} )
legend location NorthWest
legend boxoff
```

```
linkaxes(hax2)
```



# Deploy a SimBiology Model

This example shows how to deploy a graphical application that simulates a SimBiology model. The example model is the Lotka-Volterra reaction system as described by Gillespie [1], which can be interpreted as a simple predator-prey model.

This example requires MATLAB Compiler<sup>™</sup>

#### Overview

You can create stand-alone SimBiology applications using the MATLAB Compiler and the SimBiology exported model. To make your application compatible with the MATLAB Compiler, do the following:

- Create an exported model, using the model's export method.
- Accelerate the model (optional).
- Save the model to a MAT file.
- Ensure your application loads the model from the MAT file.
- Add the %#function pragma to the application's top-level function.
- Call the mcc function, explicitly adding the MAT file and the exported model's dependent files to the application.

#### Load the Model

sbioloadproject lotka m1

#### **Create the Exported Model**

exportedModel = export(m1);

#### Accelerate the Model

Acceleration requires a correctly configured MEX compiler (see the documentation for mex -setup).

accelerate(exportedModel);

#### Save the Exported Model

Uncomment the next line to save the model in exportedLotka.mat

```
% save exportedLotka exportedModel
```

#### **Call** mcc

The top-level function for the application, simulateLotkaGUI.m, has already been updated to use the exported model MAT file. It also contains the following %#function pragma, which tells the MATLAB Compiler that the application uses a SimBiology exported model: %#function SimBiology.export.Model

Now, determine the list of files to explicitly add to the application. This list includes the MAT file containing the exported model and any files listed in the DependentFiles property of the exported model. Note that this MAT file must be loaded into the workspace before mcc is called, so that the exported model's files are available for deployment.

```
% For the purposes of this example, we programmatically construct the |mcc|
% command:
mccCommand = ['mcc -m simulateLotkaGUI.m -a exportedLotka.mat ' ...
sprintf(' -a %s', exportedModel.DependentFiles{:})];
% Uncomment the following line to execute the |mcc| command. This may take
% several minutes.
%
% eval(mccCommand)
```

#### References

[1] Gillespie D.T. "Exact Stochatic Simulation of Coupled Chemical Reactions," (1977) *The Journal of Physical Chemistry*, 81(25), 2340-2361.

# A

# **Creating Reaction Rates**

- "Defining Reaction Rates with Mass Action Kinetics" on page A-2
- "Defining Reaction Rates with Enzyme Kinetics" on page A-8

# **Defining Reaction Rates with Mass Action Kinetics**

# **Definition of Mass Action Kinetics**

Mass action describes the behavior of reactants and products in an elementary chemical reaction. Mass action kinetics describes this behavior as an equation where the velocity or rate of a chemical reaction is directly proportional to the concentration of the reactants.

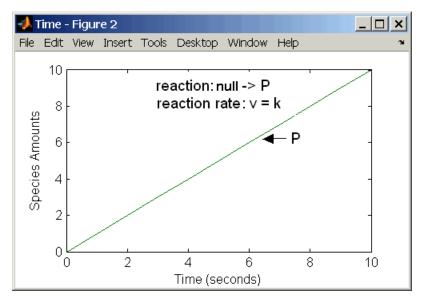
# **Zero-Order Reactions**

With a zero-order reaction, the reaction rate does not depend on the concentration of reactants. Examples of zero-order reactions are synthesis from a null species, and modeling a source species that is added to the system at a specified rate.

```
reaction: null -> P
reaction rate: k mole/second
    species: P = 0 mole
    parameters: k = 1 mole/second
```

**Note** When specifying a null species, the reaction rate must be defined in units of amount per unit time not concentration per unit time.

Entering the reaction above into the software and simulating produces the following result:



**Zero-Order Mass Action Kinetics** 

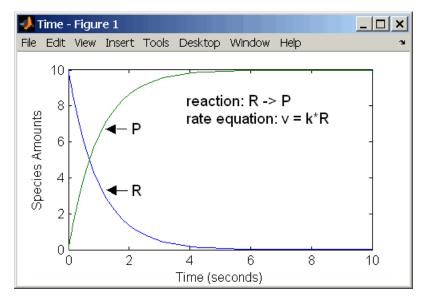
**Note** If the amount of a reactant with zero-order kinetics reaches zero before the end of a simulation, then the amount of reactant can go below zero regardless of the solver or tolerances you set.

# **First-Order Reactions**

With a first-order reaction, the reaction rate is proportional to the concentration of a single reactant. An example of a first-order reaction is radioactive decay.

```
reaction: R -> P
reaction rate: k*R mole/(liter*second)
    species: R = 10 mole/liter
    P = 0 mole/liter
    parameters: k = 1 1/second
```

Entering the reaction above into the software and simulating produces the following results:



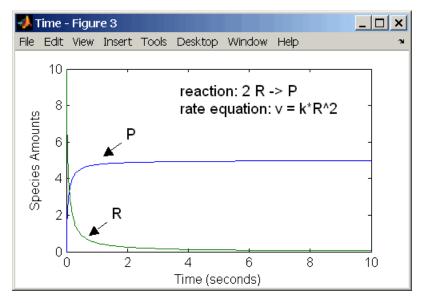
**First-Order Mass Action Kinetics** 

# **Second-Order Reactions**

A second-order reaction has a reaction rate that is proportional to the square or the concentration of a single reactant or proportional to two reactants. Notice the space between the reactant coefficient and the name of the reactant. Without the space, 2R would be considered the name of a species.

```
reaction: 2 R -> P
reaction rate: k*R^2 mole/(liter*second)
    species: R = 10 mole/liter
    P = 0 mole/liter
    parameters: k = 1 liter/(mole*second)
```

Entering the reaction above into the software and simulating produces the following results:

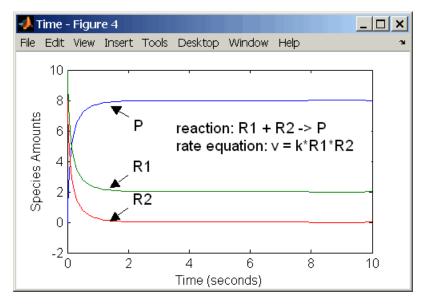


Second-Order Kinetics with Single Reactant

With two reactants, the reaction rate depends on the concentration of two of the reactants.

```
reaction: R1 + R2 -> P
reaction rate: k*R1*R2 mole/(liter*second)
    species: R1 = 10 mole/liter
        R2 = 8 mole/liter
        P = 0 mole/liter
        parameters: k = 1 liter/(mole*second)
```

Enter the reaction above into the software and simulating produces the following results. There is a difference in the final values because the initial amount of one of the reactants is lower than the other. After the first reactant is used up, the reaction stops.



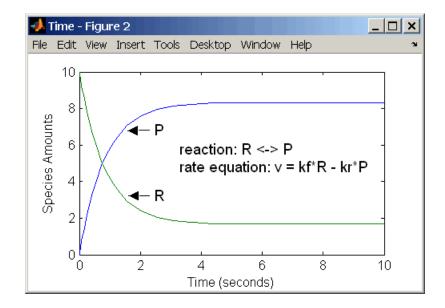
Second-Order Kinetics with Two Reactants

# **Reversible Mass Action**

You can model reversible reactions with two separate reactions or with one reaction. With a single reversible reaction, the reaction rates for the forward and reverse reactions are combined into one expression. Notice the angle brackets before and after the hyphen to represent a reversible reaction.

```
reaction: R <-> P
reaction rate: kf*R - kr*P mole/(liter*second)
    species: R = 10 mole/liter
    P = 0 mole/liter
parameters: kf = 1 1/second
    kr = 0.2 1/second
```

Entering the reaction above into the software and simulating produces the following results. At equilibrium when the rate of the forward reaction equals the reverse reaction, v = kf\*R - kr\*P = 0 and P/R = kf/kr.



# **Defining Reaction Rates with Enzyme Kinetics**

# Simple Model for Single Substrate Catalyzed Reactions

A simple model for enzyme-catalyzed reactions starts a substrate S reversibly binding with an enzyme E. Some of the substrate in the substrate/enzyme complex is converted to product P with the release of the enzyme.

 $\begin{array}{l} S+E \xrightarrow{k1} ES \xrightarrow{k2} E+P \\ v_1 = k_1[S][E], \quad v_{1r} = k_{1r}[ES], \quad v_2 = k_2[ES] \end{array}$ 

This simple model can be defined with

- Differential rate equations. See "Enzyme Reactions with Differential Rate Equations" on page A-8.
- Reactions with mass action kinetics. See "Enzyme Reactions with Mass Action Kinetics" on page A-10.
- Reactions with Henri-Michaelis-Menten kinetics. See "Enzyme Reactions with Irreversible Henri-Michaelis-Menten Kinetics" on page A-11.

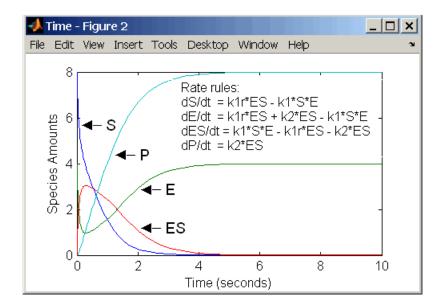
## **Enzyme Reactions with Differential Rate Equations**

The reactions for a single-substrate enzyme reaction mechanism (see "Simple Model for Single Substrate Catalyzed Reactions" on page A-8) can be described with differential rate equations. You can enter the differential rate equations into the software as rate rules.

```
reactions: none
reaction rate: none
   rate rules: dS/dt
                    = k1r*ES - k1*S*E
               dE/dt
                    = k1r*ES + k2*ES - k1*S*E
               dES/dt = k1*S*E - k1r*ES - k2*ES
               dP/dt = k2*ES
      species: S = 8
                       mole
               E = 4
                       mole
              ES = 0
                       mole
               P = 0
                       mole
```

```
parameters: k1 = 2 1/(mole*second)
k1r = 1 1/second
k2 = 1.5 1/second
```

Remember that the rate rule dS/dt = f(x) is written in a SimBiology rate rule expression as S = f(x). For more information about rate rules see "Rate Rules" on page 1-28.



Alternatively, you could remove the rate rule for ES, add a new species Etotal for the total amount of enzyme, and add an algebraic rule 0 = Etotal - E - ES, where the initial amounts for Etotal and E are equal.

```
reactions: none
reaction rate: none
rate rules: dS/dt = k1r*ES - k1*S*E
dE/dt = k1r*ES + k2*ES - k1*S*E
dP/dt = k2*ES
algebraic rule: 0 = Etotal - E - ES
species: S = 8 mole
E = 4 mole
ES = 0 mole
```

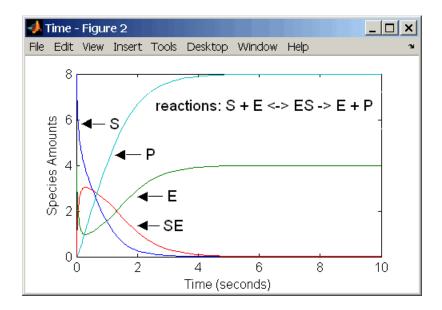
```
P = 0 mole
Etotal = 4 mole
parameters: k1 = 2 1/(mole*second)
k1r = 1 1/second
k2 = 1.5 1/second
```

## **Enzyme Reactions with Mass Action Kinetics**

Determining the differential rate equations for the reactions in a model is a time-consuming process. A better way is to enter the reactions for a single substrate enzyme reaction mechanism directly into the software. The following example using models an enzyme catalyzed reaction with mass action kinetics. For a description of the reaction model, see "Simple Model for Single Substrate Catalyzed Reactions" on page A-8.

```
reaction: S + E -> ES
reaction rate: k1*S*E (binding)
     reaction: ES -> S + E
reaction rate: k1r*ES (unbinding)
     reaction: ES -> E + P
reaction rate: k2*ES (transformation)
      species: S = 8
                        mole
               E = 4
                       mole
              ES = 0
                        mole
               P = 0
                        mole
                       1/(mole*second)
   parameters: k1 = 2
               k1r = 1
                        1/second
               k2 = 1.5 1/second
```

The results for a simulation using reactions are identical to the results from using differential rate equations.



## Enzyme Reactions with Irreversible Henri-Michaelis-Menten Kinetics

Representing an enzyme-catalyzed reaction with mass action kinetics requires you to know the rate constants k1, k1r, and k2. However, these rate constants are rarely reported in the literature. It is more common to give the rate constants for Henri-Michaelis-Menten kinetics with the maximum velocity Vm=k2\*E and the constant Km = (k1r + k2)/k1. The reaction rate for a single substrate enzyme reaction using Henri-Michaelis-Menten kinetics is given below. For information about the model, see "Simple Model for Single Substrate Catalyzed Reactions" on page A-8.

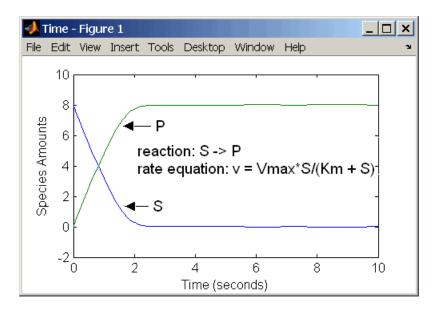
 $v = \frac{Vmax[S]}{Km + [S]}$ 

The following example models an enzyme catalyzed reaction using Henri-Michaelis-Menten kinetics with a single reaction and reaction rate equation. Enter the reaction defined below into the software and simulate.

```
reaction: S -> P
reaction rate: Vmax*S/(Km + S)
```

species: S = 8 mole P = 0 mole parameters: Vmax = 6 mole/second Km = 1.25 mole

The results show a plot slightly different from the plot using mass action kinetics. The differences are due to assumptions made when deriving the Michaelis-Menten rate equation.



# **Creating Rate Rules**

- "Using Rate Rules When the Rate of Change Is Constant" on page B-2
- "Using Rate Rules When the Rate of Change Is Exponential" on page B-4
- "Using Rate Rules When the Rate of Change Is Determined by Another Species" on page B-6
- "Using Rate Rules To Express Differential Rate Equations as Rules" on page B-8

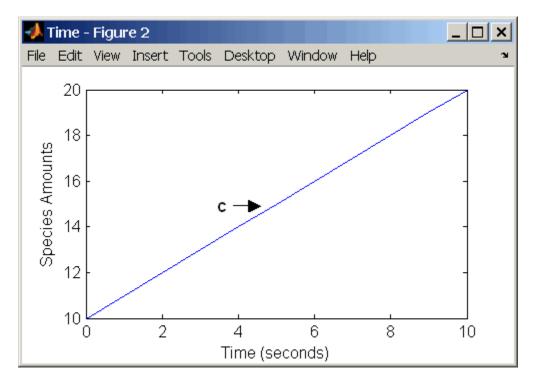
# Using Rate Rules When the Rate of Change Is Constant

You can increase or decrease the amount or concentration of a species by a constant value using a zero-order rate rule. For example, suppose species c increases by a constant rate k.

```
reaction: none
rate equation: none
rate rule: dc/dt = k
species : c = 10 mole(initial amount)
parameters: k = 1 mole/second
```

The analytical solution is  $c = kt + c_0$ , where  $c_0$  is the initial amount or concentration of the species c.

Enter the rule described above as c = k. Set the RuleType property to rate, enter the values for c and k, and then simulate.



# Using Rate Rules When the Rate of Change Is Exponential

You can change the amount of a species similar to a first-order reaction using a first-order rate rule. For example, suppose the species **c** decays exponentially.

The solution for the rate rule dc/dt = -k\*c is  $c = c_0 e^{-kt}$ .

Time - Figure 1 File Edit View Insert Tools Desktop Window Help 10 8 Species Amounts 6 С 4 2 0 2 4 8 6 0 10 Time (seconds)

Enter the rate rule described above and the simulate.

Notice that if the amount of a species c is determined by a rate rule and c is also in a reaction, c must have its BoundaryCondition property set to true. For example, with a reaction  $a \rightarrow c$  and a rate rule dc/dt = k\*c, set the BoundaryCondtion property for c so that a differential rate term is not created from the reaction. The amount of c is determined solely by a differential rate term from the rate rule.

If the BoundaryCondtion property is set to  ${\tt false},$  you will get the following error message:

Invalid rule variable 'in a reaction or another rule'.

# Using Rate Rules When the Rate of Change Is Determined by Another Species

A species from one reaction can determine the rate of another reaction if it is in the second reaction rate equation. Similarly, a species from a reaction can determine the rate of another species if it is in the rate rule that defines that other species.

```
reaction: a -> b
rate equation: v = -k1*a
rate rule: dc/dt = k2*a
species: a = 10 mole
b = 0 mole
c = 5 mole
parameters: k1 = 1 1/second
k2 = 1 1/second
```

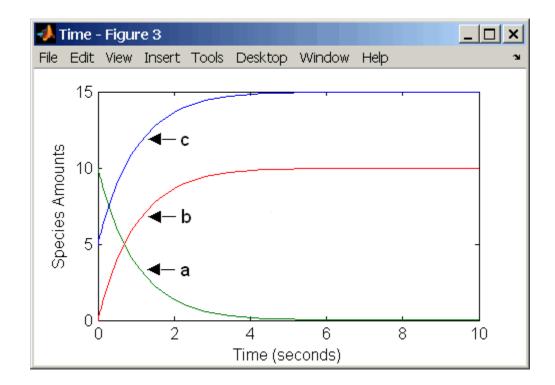
The solution for the species in the reaction are:

 $a=a_0e^{-k1t}$  and  $b=a_0(1-e^{-k1t})$ 

With the rate rule  $dc/dt = k_2 * a$  dependent on the reaction,  $dc/dt = k_2(a_0e^{-kt})$ , and the solution is:

$$c = c_0 + k_2 a_0 / k_1 (1 - e^{-k_1 t})$$

Enter the reaction and rule described above and simulate.



# Using Rate Rules To Express Differential Rate Equations as Rules

Many mathematical models in the literature are described with differential rate equations for the species. You could manually convert the equations to reactions, or you could enter the equations as rate rules. For example, you could enter the following differential rate equation for a species C:

$$\frac{dC}{dt} = vi - vdX \frac{C}{Kc + C} - kdC$$

as a rate rule in SimBiology:

C = vi - (vd\*X\*C)/(Kc + C) - kd\*C

# Models Used in Examples

- "Minimal Cascade Model for a Mitotic Oscillator" on page C-2
- "Model of the Yeast Heterotrimeric G Protein Cycle" on page C-19
- "Model of M-Phase Control in Xenopus Oocyte Extracts" on page C-25

# Minimal Cascade Model for a Mitotic Oscillator

Albert Goldbeter modified a model with enzyme cascades [Goldbeter and Koshland 1981] to fit cell cycle data from studies with embryonic cells [Goldbeter 1991]. He used this model to demonstrate thresholds with enzyme cascades and periodic behavior caused by negative feedback.

There are two SimBiology model variations using Goldbeter's model. The first model uses the differential rate equations directly from Goldbeter's paper. The second model is built with reactions using Henri-Michaelis-Menten kinetics.

#### In this section...

"Goldbeter Model" on page C-2

"SimBiology Model with Rate Rules" on page C-5

"SimBiology Model with Reactions" on page C-7

"References" on page C-18

# **Goldbeter Model**

- "About the Goldbeter Model" on page C-2
- "Reaction Descriptions and Model Assumptions" on page C-3
- "Mathematical Model" on page C-4

#### About the Goldbeter Model

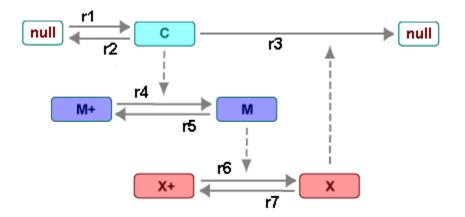
Albert Goldbeter created a simple cell division model from studies with embryonic cells [Goldbeter 1991]. This model demonstrates thresholds with enzyme cascades and periodic behavior caused by negative feedback.

There are six species in Goldbeter's minimal mitotic oscillator model [Goldbeter 1991].

• C — Cyclin. The periodic behavior of cyclin activates and deactivates an enzyme cascade.

- M+, M Inactive (phosphorylated) and active forms of cdc2 kinase. Kinases catalyze the addition of phosphate groups onto amino acid residues.
- X+, X Inactive and active (phosphorylated) forms of a cyclin protease. Proteases degrade proteins by breaking peptide bonds.

The reactions are labeled r1 to r7 on the following diagram.



This model shows:

- How thresholds with cdc2 kinase activation (M + -> M) and protease activation (X + -> X) can occur as the result of covalent modification (for example, phosphorylation or dephosphorylation), but without the need for positive feedback.
- How periodic behavior with cdc2 kinase activation can occur with negative feedback and the time delay associated with activation/deactivation enzyme cascades.

### **Reaction Descriptions and Model Assumptions**

The following list describes each of the reactions in Goldbeter's minimal mitotic oscillator with some of the simplifying assumptions. For a more detailed explanation of the model, see [Goldbeter 1991].

С

- Cyclin (C) is synthesized at a constant rate (r1) and degraded at a constant rate (r2).
- Cyclin (C) does not complex with cdc2 kinase (M).
- Cyclin (C) activates cdc2 kinase (M+ -> M) by increasing the velocity of the phosphatase that activates the kinase. Inactive cdc2 kinase (M+) is activated by removing inhibiting phosphate groups (r4).
- The amount of deactivating kinase (not modeled) for the cdc2 kinase (M) is constant. Active cdc2 kinase (M) is deactivated by adding inhibiting phosphate group (r5).
- The activation of cyclin protease (X+ -> X) by the active cdc2 kinase (M) is direct without other intervening cascades. Cyclin protease (X) is activated by adding phosphate groups (r6).
- The amount of deactivating phosphatase (not modeled) for the cyclin protease (X) is constant. Active cyclin protease (X) is deactivated by removing the activating phosphate groups (r7).
- The three species of interest are cyclin (C), active dephosphorylated cdc2 kinase (M), and active phosphorylated protease (X). The total amounts of (M + M+) and (X + X+) are constant.

### **Mathematical Model**

Goldbeter's minimal mitotic oscillator model is defined with three differential rate equations and two algebraic equations that define changing parameters in the rate equations.

**Differential Rate Equation 1, Cyclin (C).** The following differential rate equation is from [Goldbeter 1991] for cyclin (C).

$$\frac{dC}{dt} = v_i - v_d X \frac{C}{K_d + C} - k_d C$$

**Differential Rate Equation 2, Kinase (M).** The following differential rate equation is for cdc2 kinase (M). Notice that (1 - M) is the amount of inactive (phosphorylated) cdc2 kinase (M+).

$$\begin{split} &\frac{dM}{dt} = V_1 \frac{(1-M)}{K_1 + (1-M)} - V_2 \frac{M}{K_2 + M} \\ &V_1 = \frac{VM_1[C]}{K_c + [C]} \end{split}$$

**Differential Rate Equation 3, Protease (X).** Differential rate equations for cyclin protease (X). Notice that (1 - X) is the amount of inactive (unphosphorylated) cyclin protease (X+).

$$\frac{dX}{dt} = V_3 \frac{(1-X)}{K_3 + (1-X)} - V_4 \frac{X}{K_4 + X}$$
$$V_3 = VM_3[M]$$

# SimBiology Model with Rate Rules

- "SimBiology Model with Rules" on page C-5
- "SimBiology Simulation with Rules" on page C-6

### **SimBiology Model with Rules**

In the literature, many biological models are defined using differential rate and algebraic equations. With SimBiology software, you can enter the equations directly as SBML rules. The example in this section uses Goldbeter's mitotic oscillator to illustrate this point.

Writing differential rate equations in an unambiguous format that a software program can understand is a fairly simple process.

- Use an asterisk to indicate multiplication. For example, k[a] is written k\*a.
- Remove square brackets that indicate concentration from around species. The units associated with the species will indicate concentration (moles/liter) or amount (moles, molecules).

SimBiology software uses square brackets around species and parameter name to allow names that are not valid MATLAB variable names. For example, you could have a species named glucose-6-phosphate dehydrogenase but you need to add brackets around the name in reaction rate and rule equations.

Use parentheses to clarify the order of evaluation for mathematical operations. For example, do not write a Henri-Michaelis-Menten rate as Vm\*C/Kd + C, because Vm\*C is divided by Kd before adding C, and then C is added to the result.

The following equation is the rate rule for "Differential Rate Equation 1, Cyclin (C)" on page C-4:

dC/dt = vi - (vd\*X\*C)/(Kd + C) - kd\*C

The following equations are the rate and repeatedAssignment rules for "Differential Rate Equation 2, Kinase (M)" on page C-4:

```
dM/dt = (V1*Mplus)/(K1 + Mplus) - (V2*M)/(K2 + M)
V1 = (VM1*C)/(Kc + C)
Mplus = Mt - M
```

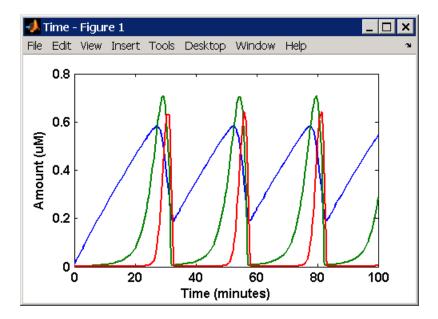
The following equations are the rate and repeatedAssignment rules for "Differential Rate Equation 3, Protease (X)" on page C-5:

```
dX/dt = (V3*Xplus)/(K3 + Xplus) - (V4*X)/(K4 + X)
V3 = VM3*M
Xplus = Xt - X
```

**Rules.** The active (M) and inactive (Mplus) forms of the kinase are assumed to be part of a conserved cycle with the total concentration (Mt) remaining constant during the simulation. You need only one differential rate equation with a mass balance equation to define the amounts of both species. Similarly, the active (X) and inactive (Xplus) forms of the protease are part of a second conserved cycle.

#### **SimBiology Simulation with Rules**

This is a simulation of Goldbeter's minimal mitotic oscillator using differential rate and algebraic equations. Simulate with the sundials solver and plot



species C, M, and X. For a description of the model, see "SimBiology Model with Rules" on page C-5.

# **SimBiology Model with Reactions**

- "Converting Differential Rate Equations to Reactions" on page C-7
- "Calculating Initial Values for Reactions" on page C-10
- "SimBiology Simulation with Reactions" on page C-17

### **Converting Differential Rate Equations to Reactions**

In the literature, many models are defined with differential rate equations. With SimBiology software, creating the differential equations from reactions is unnecessary; you can enter the reactions and let the software calculate the equations.

Some models are defined with differential rate equations, and you might need the reactions to be compatible with your model. Two rules you can use to convert differential rate equations to reactions are: С

- For a positive term The species described by the equation is placed on the right as a product, and the species in the term are placed on the left as reactants.
- For a negative term The species described by the equation is placed on the left as a product, and the species in the term are also placed on the left as reactants.

You need to determine the products using additional information, for example, a reaction diagram, a description of the model, or an understanding of a reaction. If a reaction is catalyzed by a kinase, then you can conclude that the product has one or more additional phosphate groups.

A simple first-order reaction has differential rate equation dR/dt = +kr[P]- kf[R]. The negative term implies that the reaction is  $R \rightarrow ?$  with an unknown product. The positive term identifies the product and completes the reaction, R <-> P.

**Reactions R1 to R3 from Equation E1.** The differential rate equation 1 is repeated here for comparison with the reactions. See "Differential Rate Equation 1, Cyclin (C)" on page C-4.

$$\frac{dC}{dt} = v_i - v_d X \frac{C}{K_d + C} - k_d C$$

The reaction and reaction rate equations from the differential rate equation E1 are given below:

```
r1 reaction: null -> C
reaction rate: vi
r2 reaction: C -> null
reaction rate: kd*C
r3 reaction: C -> null
reaction rate: (vd*X*C)/(Kd + C)
```

**Reactions R4 and R5 from Equation E2.** The differential rate equation 2 and algebraic equation 2 are repeated here for comparison with the reactions. See "Differential Rate Equation 2, Kinase (M)" on page C-4.

$$\begin{split} &\frac{dM}{dt} = V_1 \frac{(1-M)}{K_1 + (1-M)} - V_2 \frac{M}{K_2 + M} \\ &V_1 = \frac{VM_1[C]}{K_c + [C]} \end{split}$$

The reaction and reaction rate equations from the differential rate equation E2 are given below:

```
r4 reaction: Mplus -> M
reaction rate: V1*Mplus/(K1 + Mplus)
repeatedAssignment rule: V1 = VM1*C/(Kc + C)
r5 reaction: M -> Mplus
reaction rate: V2*M/(K2 + M)
```

**Reactions R6 and R7 from Equation E3.** The differential rate equation for equation 3 and algebraic equation 3 is repeated here for comparison with the reactions.

$$\frac{dX}{dt} = V_3 \frac{(1-X)}{K_3 + (1-X)} - V_4 \frac{X}{K_4 + X}$$
 V3 = VM3\*[M]

The reaction and reaction rate equations from the differential rate equation E3 are given below:

```
r6 reaction: Xplus -> X
reaction rate: V3*Xplus]/(K3 + Xplus)
repeatedAssignment rule: V3 = VM3*M
r7 reaction: X -> Xplus
reaction rate: V4*X/(K4 + X)
```

### **Calculating Initial Values for Reactions**

After you converted the differential rate equations to the reactions and reaction rate equations, you can start to fill in initial values for the species (reactants and products) and parameters.

The initial values for parameters and amounts for species are listed with four different units in the same dimension:

- A Original units in the Goldbeter 1991 paper.
- B Units of concentration with time converted to second. When converting a to b, use 1 minute = 60 second for parameters.

	X uM	1e-6 mole/liter	1 minute	Y mole
	minute	1 uM	60 second	liter*second
C	Unite of	amount og molog	Whon cont	orting concentration

• C — Units of amount as moles. When converting concentration to moles, use a cell volume of 1e-12 liter and assume that volume does not change.

Y mole	v	1e-12 liter	_	Z mole
liter*second	А		-	second

• D — Units of amount as molecules. When converting amount as moles to molecules, use 6.022e23 molecules = 1 mole.

Z mole	6.022e23 molecule	N molecules
second	1 mole	second

With dimensional analysis on and unit conversion off, select all of the units for one letter. For example, select all of the As. If dimensional analysis and unit conversion are on, you can mix and match letters and get the same answer.

#### **Reaction 1 Cyclin Synthesis.**

R1		Value	Units
reaction	null -> C		
reaction rate	vi		A. uM/minute
			B. mole/(liter*second)

R1		Value	Units
			C. mole/second
			D. molecule/second
parameters	vi	0.025	A. uM/minute
		4.167e-10	B. mole/(liter*second)
		4.167 e- 22	C. mole/second
		205	D. molecule/second
species	С	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

## Reaction 2 Cyclin Undifferentiated Degradation.

R2		Value	Units
reaction	C -> null		
reaction rate	kd*C		A. uM/minute
			B. mole/(liter*second)
			C. mole/second
			D. molecule/second
parameters	kd	0.010	A. 1/minute
		1.6667 e-4	B, C, D. 1/second
species	C	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

R3		Value	Units
reaction	C -> null		
reaction rate	(vd*X*C)/(Kd + C)		A. uM/minute
			B. mole/(liter*second)
			C. mole/second
			D. molecule/second
parameter	vd	0.25	A. 1/minute
		0.0042	B, C, D. 1/second
parameter	Kd	0.02	A. uM
		2.0e-8	B. mole/liter
		2.0e-020	C. mole
		12044	D. molecule
species	C (substrate)	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule
species	X (enzyme)	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

## **Reaction 3 Cyclin Protease Degradation.**

## Reaction 4 Cdc2 Kinase Activation.

R4		Value	Units
reaction	Mplus -> M		
reaction rate	(V1*Mplus)/(K1 + Mplus)		A. uM/minute

R4		Value	Units
			B. mole/(liter*second)
			C. mole/second
			D. molecule/second
repeatedAssignme rule	en¥1 = (VM1*C)/(Kc + C)		
parameter	V1 (variable by rule)	0.00	A. uM/minute
			B. mole/(liter*second)
			C. mole/second
			D. molecule/second
parameter	VM1	3.0	A. uM/minute
		5.0e-8	B. mole/(liter*second)
		5.0000e-020	C. mole/second
		30110	D. molecule/second
parameter	Кс	0.5	A. uM
		5.0000e-7	B. mole/liter
		5.0e-19	C. mole
		3.011e+5	D. molecule
parameter	K1	0.005	A. uM
		5e-9	B. mole/liter
		5e-21	C. mole
		3.011e+3	D. molecule
species	Mplus (inactive substrate)	0.99	A. uM
		9.9e-7	B. mole/liter
		9.9e-19	C. mole
		5.962e + 5	D. molecule
species	M (active product)	0.01	A. uM

R4		Value	Units
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule
species	C	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

## Reaction 5 Cdc2 Kinase Deactivation.

R5		Value	Units
reaction	M -> M_plus		
reaction rate	(V2*M)/(K2 + M)		A. uM/minute
			B. (mole/liter-second)
			C. mole/second
			D. molecule/second
parameter	V2	1.5	A. uM/minute
		2.5000e-008	B. mole/liter-second
		2.5000e-020	C. mole/second
		15055	D. molecule/second
parameter	К2	0.005	A. uM
		5.0000e-009	B. mole/liter
		5.0000e-021	C. mole
		3011	D. molecule
		1.0e-20	C. mole
species	Mplus (inactive)	0.99	A. uM
		9.9e-7	B. mole/liter

R5		Value	Units
		9.9e-19	C. mole
		5.962e+5	D. molecule
species	M (active)	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

## Reaction 6 Protease Activation.

	Value	Units
Xplus -> X		
(V3*Xplus)/(K3 + Xplus)		A. uM/minute
		B. mole/(liter*second)
		C. mole/second
		D. molecule/second
repeatedAssignme <b>№8</b> = VM3*M rule		
V3 (variable by rule)		A. uM/minute
		B. mole/liter-second
		C. mole/second
		D. molecule/second
VM3	1.0	A. 1/minute
	0.0167	B, C, D. 1/second
К3	0.005	A. uM
	5e-9	B. mole/liter
	5e-21	C. mole
	3.011e+3	D. molecule
	(V3*Xplus)/(K3 + Xplus) eN/B = VM3*M V3 (variable by rule) VM3	<pre>Xplus -&gt; X (V3*Xplus)/(K3 + Xplus)    * * * *</pre>

Ró		Value	Units
species	Xplus (inactive substrate)	0.99	A. uM
		9.9e-7	B. mole/liter
		9.9e-19	C. mole
		5.962e + 5	D. molecule
species	X (active product)	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule
species	M (enzyme)	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

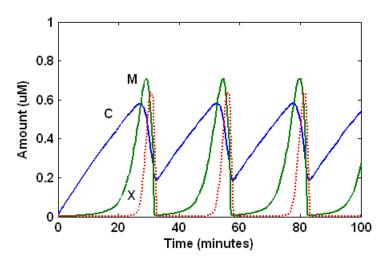
## Reaction 7 Protease Deactivation.

R7		Value	Units
reaction	X -> X_plus		
reaction rate	(V4*X)/(K4 + X)		A. uM/minute
			B. mole/(liter*second)
			C. mole/second
			D. molecule/second
parameter	V4	0.5	A. uM/minute
		8.3333e-009	B. mole/(liter*second)
		8.3333e-021	C. mole/second
		5.0183e+003	<sup>3</sup> D. molecule/second
parameter	K4	0.005	A. uM
		5e-9	B. mole/liter

R7		Value	Units
		5e-21	C. mole
		3011	D. molecule
species	Xplus (inactive)	0.99	A. uM
		9.9e-7	B. mole/liter
		9.9e-19	C. mole
		5.962e+5	D. molecule
species	X (active)	0.01	A. uM
		1e-8	B. mole/liter
		1.0e-20	C. mole
		6.022e+3	D. molecule

## **SimBiology Simulation with Reactions**

This is a simulation of Goldbeter's minimal mitotic oscillator with rate and algebraic equations. Simulate with the sundials solver and plot species C, M, and X. For a description of the model, see "SimBiology Model with Reactions" on page C-7.



## References

[1] Goldbeter A. (1991), "A minimal cascade model for the mitotic oscillator involving cyclin and cdc2 kinase," Proceedings of the National Academy of Sciences USA, 88:9107-9111.

[2] Goldbeter A., Koshland D. (1981), "An amplified sensitivity arising from covalent modification in biological systems," Proceedings of the National Academy of Sciences USA, 78:6840-6844.

[3] Goldbeter A., Koshland D. (1984), "Ultrasensitivity in biochemical systems controlled by covalent modification," The Journal of Biological Chemistry, 259:14441-14447.

[4] Goldbeter A., home page on the Web, http://www.ulb.ac.be/sciences/utc/GOLDBETER/agoldbet.html

[5] Murray A.W., Kirschner M.W. (1989), "Cyclin synthesis drives the early embryonic cell cycle," Nature, 339:275-280.

# Model of the Yeast Heterotrimeric G Protein Cycle

#### In this section...

"Background on G Protein Cycles" on page C-19

"Modeling a G Protein Cycle" on page C-20

"References" on page C-24

## **Background on G Protein Cycles**

- "G Proteins" on page C-19
- "G Proteins and Pheromone Response" on page C-20

#### **G** Proteins

Cells rely on signal transduction systems to communicate with each other and to regulate cellular processes. G proteins are GTP-binding proteins that are involved in the regulation of many cellular processes. There are two known classes of G proteins: the monomeric G proteins (one GTPase), and the heterotrimeric G proteins (three different monomers). The G proteins usually facilitate a step requiring energy. This energy is supplied by the hydrolysis of GTP by a GTPase activating protein (GAP). The exchange of GDP for GTP is catalyzed by a guanine nucleotide releasing protein (GNRP) [Alberts et al. 1994].

$$Gprotein + GTP \xrightarrow{GAP} Gprotein + GDP$$

G protein-coupled receptors (GPCRs) are the targets of many pharmaceutical agents. Some estimates suggest that 40 to 50% of currently marketed drugs target GPCRs and that 40% of current drug discovery focus is on GPCR targets. Some examples include those for reducing stomach acid (ranitidine which targets histamine H2 receptor), migraine (sumatriptan, which targets a serotonin receptor subtype), schizophrenia (olanzapine, which targets serotonin and dopamine receptors), allergies (desloratadine, which targets histamine receptors). One approach in pharmaceutical research is to model

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signaling pathways to analyze and predict both downstream effects and effects in related pathways. This tutorial examines model building and analysis of the G protein cycle in the yeast pheromone response pathway using the SimBiology desktop.

#### **G** Proteins and Pheromone Response

In the yeast *Saccharomyces cerevisiae*, G protein signaling in pheromone response is a well characterized signal transduction pathway. The pheromone secreted by *alpha* cells activates the G protein-coupled  $\alpha$ -factor receptor (Ste2p) in *a* cells which results in a variety of cell responses including cell-cycle arrest and synthesis of new proteins. The authors of the study performed a quantitative analysis of this cycle, compared the regulation of G protein activation in wild-type yeast haploid *a* cells with cells containing mutations that confer supersensitivity to  $\alpha$ -factor. They analyzed the data in the context of cell-cycle arrest and pheromone-induced transcriptional activation and developed a mathematical model of the G protein cycle that they used to estimate rates of activation and deactivation of active G protein in the cell.

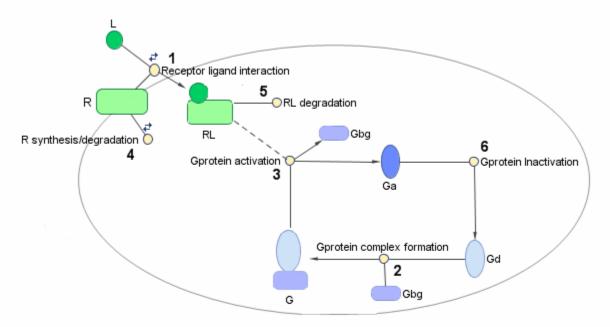
## Modeling a G Protein Cycle

- "Reactions Overview" on page C-20
- "Assumptions, Experimental Data, and Units in the G Protein Model" on page C-22

#### **Reactions Overview**

Systems biologists represent biological pathways and processes as reactions with reaction rates, and treat the components of these pathways as individual species.

The G protein cycle in the yeast pheromone-response pathway can be condensed into a set of biochemical reactions. These reactions are complex formation, transformation, or disassociation reactions that Yi and colleagues [Yi et al. 2003] use to simplify and describe the system. In this example,  $\alpha$ -factor,  $\alpha$ -factor receptor, and the G protein subunits are all treated as species participating in reactions. The system can be graphically represented as follows.



Graphical Representation of the G protein cycle in yeast pheromone response. The numbers represent reaction numbers referenced in the text. L = Ligand (alpha factor), R = alpha-factor receptor, Gbg = free levels of G-beta:G-gamma complex, Ga = active G-alpha-GTP, Gd = inactive G-alpha-GDP, G = inactive Gbg:Gd complex.

The following table shows you the reactions used to model the G protein cycle and the corresponding rate constants (rate parameters) for each reaction. For reversible reactions, the forward rate parameter is listed first.

No	. Name	Reaction	Rate Parameters
1	Receptor-ligand interaction	L + R <-> RL	kRL, kRLm
2	Heterotrimeric G protein formation	Gd + Gbg -> G	kG1
3	G protein activation	RL + G -> Ga + Gbg + RL	kGa

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No	. Name	Reaction	Rate Parameters
4	Receptor synthesis and degradation	R <-> null	kRdo, kRs
5	Receptor-ligand degradation	RL -> null	kRD1
6	G protein inactivation	Ga -> Gd	kGd

Note that in reaction 3 (G protein activation), RL appears on both sides of the reaction. This is because RL is treated as a modifier or catalyst, and the model assumes that there is no synthesis or consumption of RL in this reaction.

The authors use a set of ordinary differential equations (ODEs) to describe the system. In the software, you can represent the biological pathway as a system of biochemical reactions and the software creates the ODEs for you. Alternatively, if you have a set of ODEs that describe your system you can enter these as rate rules. For an example of modeling using rate rules, see "SimBiology Model with Rate Rules" on page C-5.

#### Assumptions, Experimental Data, and Units in the G Protein Model

The authors have obtained experimental data either through their own measurements or through published literature. As with any other model, the G protein cycle model simplifies the biological process while also trying to reconcile the experimental data. Consider these points:

- Reaction 2 Binding and formation of the heterotrimeric G protein complex is treated as a single-step reaction.
- Reaction 3 Activation of G protein is modeled as a single-step. Guanine nucleotide exchange factors (GEFs) are not modeled.
- Reactions 3 and 6 The parameters for the rate of G protein activation and deactivation (kGa and kGd) have been estimated based on the dose response curves in the reference paper. The SimBiology model being built in this tutorial directly uses those values.
- Reactions 4 and 5 Receptor synthesis and degradation are handled purely as two simple reaction steps.

• Reaction 6 — Deactivation of G protein by the regulator of G protein signaling (RGS) protein Sst2p is modeled as a single step. Sst2p is not modeled.

The reaction is modeled with an estimated reaction rate of  $0.11 \text{ s}^{-1}$ ) in the Sst2p containing wild-type strain. The uncatalyzed reaction rate is estimated to be  $0.004 \text{ s}^{-1}$  in a strain with a deletion of SST2 (*sst2A*, mutant strain).

• Free GDP, GTP, and Pi are not included in the model.

This tutorial shows you how to plot the experimental data over the simulation plot of the active G protein fraction. You can estimate the values of the experimental data of interest for this example from the coordinates of the plots found in Figure 5 of the reference paper [Yi et al. 2003]. The following values were obtained by comparing the coordinates of the standards with those of the unknowns in the figure.

Time	Fraction of Active Ga (Experimental)
0	0.00
10	0.35
30	0.40
60	0.36
110	0.39
210	0.33
300	0.24
450	0.17
600	0.20

**Note** The SimBiology **Dimensional Analysis** feature is not used in this tutorial. For this tutorial, the values of all species are converted to have the unit molecule, and all rate parameters are converted to have either the unit 1/second or the units 1/(molecule\*second), depending on whether the reaction is first or second order. You should leave the **InitialAmountUnits** box for species and the **ValueUnits** box for rate parameters empty for the models in this tutorial.

## References

[1] Tau-Mu Yi, Hiroaki Kitano, and Melvin I. Simon. A quantitative characterization of the yeast heterotrimeric G protein cycle. PNAS (2003) vol. 100, 10764-10769.

[2] Alberts, B., Bray, D., Lewis, J., Raff, M., Roberts, K., and Watson, J.D. Molecular Biology of the Cell, 3rd edition, Garland Publishing, 1994.

# Model of M-Phase Control in Xenopus Oocyte Extracts

John Tyson's Computational Cell Biology Lab created a mathematical model for M-phase control in *Xenopus* oocyte (frog egg) extracts [Marlovits et al. 1998]. The M-phase control model shows principles by which you can apply phosphorylation and regulatory loops in your own models. Publications typically list systems of ordinary differential equations (ODEs) that represent a model system. This example shows you how to interpret these ODEs in the form of reaction pathways that are easier to represent and visualize in SimBiology software.

The model is centered around M-phase promoting factor (MPF). There are two positive feedback loops where MPF increases its synthesis and a negative feedback loop where MPF decreases its amount by increasing its degradation.

#### In this section...

"M-Phase Control Model" on page C-25

"M-Phase Control Equations" on page C-27

"SimBiology Model with Rate and Algebraic Rules" on page C-36

"SimBiology Model with Reactions and Algebraic Rules" on page C-43

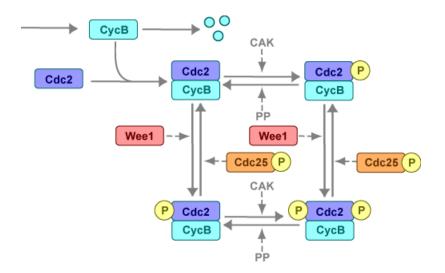
"References" on page C-60

## **M-Phase Control Model**

- "Synthesis Reactions" on page C-25
- "Regulation Reactions with Active MPF" on page C-26

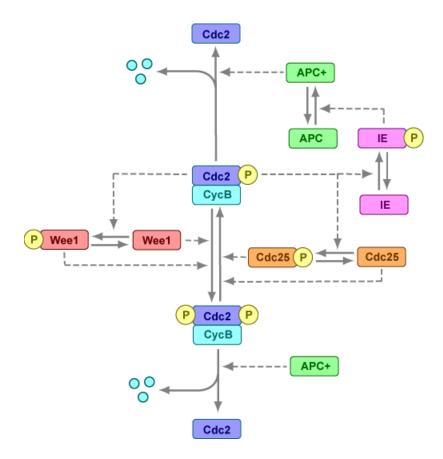
#### **Synthesis Reactions**

Cyclin B (CycB) dimerizes with Cdc2 kinase (Cdc2) to form M-phase promoting factor (MPF).



## **Regulation Reactions with Active MPF**

Positive feedback loops with M-phase promoting factor (MPF) activate the Cdc25 phosphatase and deactivate the Wee1 kinase. A negative feedback loop with MPF activates anaphase-promoting complex (APC) that regulates the degradation of the Cyclin B subunit.



# **M-Phase Control Equations**

- "About the Rate Equations in This Example" on page C-28
- "Converting Differential Equations to Reactions" on page C-28
- "Equation 1, Cyclin B" on page C-29
- "Equation 2, M-Phase Promoting Factor" on page C-30
- "Equation 3, Inhibited M-Phase Promoting Factor" on page C-31
- "Equation 4, Inhibited and Activated M-Phase Promoting Factor" on page C-31

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- "Equation 5, Activated M-Phase Promoting Factor" on page C-32
- "Equation 11, Cell Division Control 25" on page C-33
- "Equation 12, Wee1 Activation/Deactivation" on page C-33
- "Equation 13, Intermediate Enzyme Activation/Deactivation" on page C-34
- "Equation 14, APC Activation/Deactivation" on page C-34
- "Equation 17, Rate Parameter K2" on page C-35
- "Equation 18, Rate Parameter Kcdc25" on page C-36
- "Equation 19, Rate Parameter Kwee1" on page C-36

## About the Rate Equations in This Example

Models in systems biology are commonly described in the literature with differential rate equations. However, SimBiology software defines a model using reactions. This section shows you how to convert models published in the literature to a SimBiology format. The equation numbers match the published paper for this model [Marlovits et al. 1998]. Equations that are missing in the sequence involve the Cdk inhibitor (CKI) protein, which is not currently modeled in the SimBiology version.

## **Converting Differential Equations to Reactions**

The rules for writing reaction and reaction rate equations from differential rate equations include not only the equations but also an understanding of the reactions. dx/dt refers to the species the differential rate equation is defining. *kinetics* refers to the species in the reaction rate.

• Positive terms: Rate species are placed on right side of the reactions; reaction rate equation species are placed on the left.

kinetics 
$$\rightarrow \frac{dx}{dt}$$

• Negative terms: Rate species are placed on the left side of the reaction because the species are being used up in some way; reaction rate equation species are also placed on left. You need to deduce the products from additional information about the model.

kinetics or 
$$(\frac{dx}{dt}) \rightarrow$$
 products?

The following table will help you deduce the products for a reaction. In this example, by convention, phosphate groups on the right side of a species name are activating while phosphate groups on left are inhibiting.

Enzyme	Description	Reaction
wee1	Kinase, add inhibiting phosphate group	MPF —> P-MPF
cdc25	Phosphatase, remove inhibiting phosphate group	P-MPF —> MPF + P
kcak	Kinase, add activating phosphate group	MPF —> MPFp
kpp	Phosphatase, remove activating phosphate group	MPF-P> MPF + P
MPF	Kinase, add activating or inhibiting phosphate group	Wee1/Cdc25/IE -> X-P or P-X
ki	Add inhibiting Cki	Cki + MPF —> Cki:MPF
kir	Remove inhibiting Cki	Cki:MPF> Cki + MPF

#### **Equation 1, Cyclin B**

Differential rate equation for cyclin B [Marlovits et al. 1998].

 $\frac{d[\text{CycB}]}{dt} = +k1 - k2[\text{CycB}] - k3[\text{Cdc2}][\text{CycB}]$ 

Rate rule using SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Rule 1 [CycB] = k1 - K2\*[CycB] - k3\*[Cdc2]\*[CycB]

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

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Reaction 1AA -> CycBv = k1Reaction 2CycB -> AAv = K2\*[CycB]Reaction 3Cdc2 + CycB -> MPFv = k3\*[Cdc2]\*[CycB]

#### **Equation 2, M-Phase Promoting Factor**

Differential rate equation for M-phase promoting factor (MPF) [Marlovits 1998]. Note that the parameter name kcakr [Marlovits et al. 1998] is changed to kpp [Borisuk 1998] in the following reaction equations. MPF is a heterodimer of cdc2 kinase and cyclin B.

 $\frac{d[\text{MPF}]}{dt} = +k3[\text{Cdc2}][\text{CycB}] - \text{K2}[\text{MPF}]$ +kpp[MPFp] - kcak[MPF]+Kcdc25[pMPF] - Kwee1[MPF]+kir[Cki:MPF] - ki[MPF][Cki]

Rate rule using SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

```
Rule 2 MPF = kpp*MPFp - (Kwee1 + kcak + K2)*MPF +
Kcdc25*pMPF + k3*Cdc2*CycB
```

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43. A reaction name in parentheses denotes a reaction repeated in another differential rate equation.

```
(Reaction 3) Cdc2 + CycB -> MPF
                                      v = k3*[Cdc2]*[CycB]
Reaction 4
             MPF \rightarrow Cdc2 + AA
                                      v = K2*[MPF]
             MPFp -> MPF
                                      v = kpp*[MPFp]
Reaction 5
Reaction 6
             MPF -> MPFp
                                      v = kcak*[MPF]
Reaction 7
             pMPF -> MPF
                                      v = Kcdc25*[pMPF]
             MPF -> pMPF
                                      v = Kwee1*[MPF]
Reaction 8
```

#### Equation 3, Inhibited M-Phase Promoting Factor

Differential rate equation for inhibited M-phase promoting factor (pMPF) [Marlovits 1998].

$$\label{eq:main_state} \begin{split} \frac{d[\text{pMPF}]}{dt} = -\text{K2}[\text{pMPF}] \\ + \text{kpp}[\text{pMPFp}] - \text{kcak}[\text{pMPF}] \\ + \text{Kwee1}[\text{MPF}] - \text{Kcdc25}[\text{pMPF}] \\ + \text{kd}[\text{Cki:pMPF}] \end{split}$$

Rate rule using SimBiology format for the differential rate equation 3. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Rule 3 pMPF = Kwee1\*MPF - (Kcdc25 + kcak + K2)\*pMPF + kpp\*pMPFp

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

Reaction 11	pMPF -> Cdc2 + AA	v = K2*[pMPF]
Reaction 12	pMPFp -> pMPF	v = kpp*[pMPFp]
Reaction 13	pMPF -> pMPFp	v = kcak*[pMPF]
(Reaction 8)	MPF -> pMPF	v = Kwee1*[MPF]
(Reaction 7)	pMPF -> MPF	v = Kcdc25*[pMPF]

#### Equation 4, Inhibited and Activated M-Phase Promoting Factor

Differential rate equation for inhibited and activated M-phase promoting factor (pMPFp) [Marlovits 1998].

```
\label{eq:model} \begin{split} \frac{d[\text{pMPFp}]}{dt} = -\text{K2}[\text{pMPFp}] \\ +\text{kcak}[\text{pMPF}] -\text{kpp}[\text{pMPFp}] \\ +\text{Kwee1}[\text{MPFp}] -\text{Kcdc25}[\text{pMPFp}] \\ +\text{kd}[\text{Cki:pMPFp}] \end{split}
```

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Rate rule using SimBiology format for the differential rate equation. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

```
Rule 4 pMPFp = Kwee1*MPFp - (kpp + Kcdc25 + K2)*pMPFp
+ kcak*pMPF
```

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

Reaction 15	pMPFp -> Cdc2 + AA	v = K2*[pMPFp]
(Reaction 13)	pMPF -> pMPFp	v = kcak*[pMPF]
(Reaction 12)	pMPFp -> pMPF	v = kpp*[pMPFp]
Reaction 16	MPFp -> pMPFp	v = Kwee1*[MPFp]
Reaction 17	pMPFp -> MPFp	v = Kcdc25*[pMPFp]

#### Equation 5, Activated M-Phase Promoting Factor

Differential rate equation for activated M-phase promoting factor (MPFp) [Marlovits 1998].

 $\frac{d[\text{MPFp}]}{dt} = -\text{K2}[\text{MPFp}]$ +kcak[MPF] -kpp[MPFp]+Kcdc25[pMPFp] -Kwee1[MPFp]+kir[CKI:MPFp] -ki[CKI][MPFp]

Rate rule using SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Rule 5 MPFp = kcak\*MPF - (kpp + Kwee1 + K2)\*MPFp + Kcdc25\*pMPFp

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

Reaction 19 MPFp -> MPF + AA v = K2\*[MPFp]

#### Equation 11, Cell Division Control 25

Differential rate equation for activating and deactivating Cdc25 [Marlovits 1998].

 $\frac{d[\text{Cdc25p}]}{dt} = +\frac{\text{k25[MPFp][Cdc25]}}{\text{Km25+[Cdc25]}} - \frac{\text{k25r[Cdc25p]}}{\text{Km25r+[Cdc25p]}}$ 

Rate rule in SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36. Note that since there isn't a rate rule for Cdc25, its amount is written as (TotalCdc25 - Cdc25p).

```
Rule 11 Cdc25p = (k25*MPFp*(TotalCdc25 - Cdc25p))/(Km25 +
(TotalCdc25 - Cdc25p)) - (k25r*PPase*Cdc25p)/(Km25r + Cdc25p)
```

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

```
Reaction 36 Cdc25 -> Cdc25p, v = k25*[MPFp]*[Cdc25]/(Km25 + [Cdc25])
Reaction 37 Cdc25p -> Cdc25, v = k25r*[Cdc25p]/(Km25r + [Cdc25p])
```

#### Equation 12, Wee1 Activation/Deactivation

Differential rate equation for activating and deactivating Wee1 kinase [Marlovits 1998]. The kinase (MPFp) phosphorylates active Wee1 (Wee1) to its inactive form (Wee1p). The dephosphorylation of inactive Wee1 (Wee1p) is by an unknown phosphatase.

 $\frac{d[Wee1]}{dt} = -\frac{kw[MPFp][Wee1]}{Kmw + [Wee1]} + \frac{kwr[Wee1P]}{Kmwr + [Wee1P]}$ 

Rate rule in SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

reaction 38 Wee1 -> Wee1p, v = (kw\*[MPFp]\*[Wee1])/(Kmw + [Wee1])
reaction 39 Wee1p -> Wee1, v = (kwr\*[Wee1p])/(Kmwr + [Wee1p])

#### Equation 13, Intermediate Enzyme Activation/Deactivation

Differential rate equation for activating and deactivating the intermediate enzyme (IE) [Marlovits 1998]. The active kinase (MPFp) phosphorylates the inactive intermediate enzyme (IE) to its active form (IEp).

 $\frac{d[IEp]}{dt} = +\frac{kie[MPFp][IE]}{Kmie + [IE]} - \frac{kier[IEp]}{Kmier + [IEp]}$ 

Rate rule in SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

```
reaction 40 IE -> IEp, v = (kie*[MPFp]*[IE])/(Kmie + [IE])
reaction 41 IEp -> IE, v = (kier*[IEp])/(Kmier + [IEp])
```

#### Equation 14, APC Activation/Deactivation

Differential rate equation for [Marlovits 1998].

```
\frac{d[APCa]}{dt} = +\frac{kap[IEP][APCi]}{Kmap + [APCi]} - \frac{kapr[APCa]}{Kmapr + [APCa]}
```

Rate rule in SimBiology format for the differential rate equation 1. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Reaction and reaction rate equations derived from the differential rate equation. For a model using these reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

```
Reaction 42 APCi -> APCa, v = (kap*[IEp]*[APCi])/(Kmap + [APCi])
Reaction 43 APCa -> APCi, v = (kapr*[APCa])/(Kmapr + [APCa])
```

#### Equation 17, Rate Parameter K2

Algebraic equation to define the rate parameter K2 [Marlovits 1998]. Inactive APC (APCi) is catalyzed by IE (intermediate enzyme) to active APC (APCa).

k2 = V2'[APC] + V2''[APC']

Algebraic rule in SimBiology format for the algebraic equation 17. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

```
Algebraic Rule 17 V2i*(TotalAPC - APCa) + V2a*APCa - K2
```

Algebraic rule when simulating with reactions. For a model using this rule with reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43. V2' is renamed to V2i and V2 is renamed to V2a. APCi (APC) is the inactive form of the enzyme while APCa (APC') is the active form. K2 is the independent variable.

```
Algebraic Rule 1 (V2i*APCi) + (V2a*APCa) - K2
```

#### Equation 18, Rate Parameter Kcdc25

Algebraic equation to define the rate parameter Kcdc25 [Marlovits 1998]. Inactive Cdc25 (Cdc25) is phosphorylated by MPF to active Cdc25 (Cdc25p).

kcdc25 = V25'[Cdc25] + V25''[Cdc25p]

Algebraic rule in SimBiology format for the algebraic equation 18. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

Algebraic Rule 18 V25i\*(TotalCdc25 - Cdc25p) + V25a\*Cdc25p - Kcdc25

Algebraic rule when simulating with reactions. Kcdc25 is the independent variable. For a model using this rule with reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

```
Algebraic Rule 2 (V25i*Cdc25) + (V25a*Cdc25p) - Kcdc25
```

#### Equation 19, Rate Parameter Kwee1

Algebraic equation to define the rate parameter [Marlovits 1998]. Active Wee1 (Wee1) is phosphorylated by MPF to inactive Wee1 (Wee1p).

kwee1 = Vwee1'[Wee1p] + Vwee1"[Wee1]

Algebraic rule in SimBiology format for rate parameter equation 19. For a model using this rule, see "SimBiology Model with Rate and Algebraic Rules" on page C-36.

```
Algebraic Rule 19 Vwee1i*Wee1p + Vwee1a*(TotalWee1 - Wee1p) - Kwee1
```

Algebraic rule when simulating with reactions. Kwee1 is the independent variable. For a model using this rule with reactions, see "SimBiology Model with Reactions and Algebraic Rules" on page C-43.

```
Algebraic Rule 3 (Vwee1i*Wee1p) + (Vwee1a*Wee1) - Kwee1
```

## SimBiology Model with Rate and Algebraic Rules

• "Overview" on page C-37

- "Writing Differential Rate Equations as Rate Rules" on page C-37
- "Species" on page C-38
- "Parameters" on page C-39
- "Rate Rule 1, Cyclin B (CycB)" on page C-40
- "Rate Rule 2, M-Phase Promoting Factor (MPF)" on page C-41
- "Rate Rule 3, Inhibited M-Phase Promoting Factor (pMPF)" on page C-41
- "Rate Rule 4, Activated but Inhibited M-Phase Promoting Factor (pMPFp)" on page C-42
- "Rate Rule 5, Activated M-Phase Promoting Factor (MPFp)" on page C-42
- "Rate Rule 11, Activated Cdc25 (Cdc25p)" on page C-42
- "Rate Rule 12, Inhibited Wee1 (Wee1p)" on page C-42
- "Rate Rule 13, Activated Intermediate Enzyme (IEp)" on page C-42
- "Rate Rule 14, Activated APC (APCa)" on page C-42
- "Algebraic Rule 17, Rate Parameter K2" on page C-43
- "Algebraic Rule 18, Rate Parameter Kcdc25" on page C-43
- "Algebraic Rule 19, Rate Parameter Kwee1" on page C-43

#### **Overview**

There is one rate rule for each equation defining a species and one algebraic rule for each variable parameter in the M-phase control model [Marlovits 1998]. For a list and description of the equations, see "M-Phase Control Equations" on page C-27.

A basic model includes rate rules 1 to 5 and 11 to 14 with algebraic rules 17, 18, and 19.

#### Writing Differential Rate Equations as Rate Rules

Writing differential rate equations in an unambiguous format that a software program can understand is a simple process when you follow the syntax rules for programming languages.

- Use an asterisk to indicate multiplication. For example, k[A] is written k\*A or k\*[A]. The brackets around the species A do not indicate concentration.
- SimBiology uses square brackets around species and parameter name to allow names that are not valid MATLAB variable names. For example, you could have a species named glucose-6-phosphate dehydrogenase but you need to add brackets around the name in reaction rate and rule equations.

[glucose-6-phosphate dehydrogenase]

Use parentheses to clarify the order of evaluation for mathematical operations. For example, do not write Henri-Michaelis-Menten reaction rates as Vm\*C/Kd + C, because Vm\*C is divided by Kd before adding C to the result. Instead, write this reaction rate as (Vm\*C)/(Kd + C).

### **Species**

The following table lists species in the model with their initial amounts. There are three variable parameters modeled as species (K2, Kcdc25, and KWee1). You could also model the variable parameters as parameters with the property **ConstantAmount** cleared.

C-38

Name	InitialAmount 🛆	InitialAmountUnits	ConstantAmount
СусВ	0.0		
MPF	0.0	<b></b>	
pMPF	0.0		
pMPFp	0.0	<b></b>	
MPFp	0.0	<b></b>	
Cdc25p	0.0	<b></b>	
Wee1p	0.0	<b></b>	
IEp	0.0		
APCa	0.0		
Kodo25	0.0	<b>_</b>	
Kwee1	0.0	<b></b>	
K2	0.0	<b></b>	
Wee1	0.0		
TotalCdc25	1.0		
TotalWee1	1.0		V
TotalAPC	1.0		V
TotallE	1.0	<b>•</b>	
PPase	1.0		V
AntiAPC	1.0		V
Cdc2	100.0	· ·	V

#### **Parameters**

The following table lists parameters in the model with their initial values. The property **ConstantValue** is selected for all of the parameters.

Name	Value 🔻	ValueUnits	ConstantValue
Vweea	1.0	<b>•</b>	
k1	1.0	<b>_</b>	
Kmwr	1.0	<b>_</b>	
Kmapr	1.0	<b>•</b>	
Km25r	1.0	<b>•</b>	
kcak	0.64	<b></b>	
V2a	0.25	<b>•</b>	
V25a	0.17	<b>•</b>	
kier	0.15	<b>•</b>	
kapr	0.13	<b>•</b>	
kap	0.13	<b>•</b>	
kwr	0.1	<b>•</b>	
k25r	0.1	<b>•</b>	
Kmw	0.1	<b>•</b>	
Km25	0.1	<b>•</b>	
kie	0.02	<b>_</b>	
kw	0.02	<b>_</b>	
k25	0.02	<b>_</b>	
V25i	0.017	<b>_</b>	
Vweei	0.01	<b>_</b>	
Kmier	0.01	<b></b>	
Kmie	0.01		
Kmap	0.01		
V2i	0.0050		
k3	0.0050		
kpp	0.0040		

## Rate Rule 1, Cyclin B (CycB)

The rate rule is from "Equation 1, Cyclin B" on page C-29.

rate rule: CycB = k1 - K2\*CycB - k3\*Cdc2\*CycB

```
species: CycB = 0 nM
Cdc2 = 100 nM, [x]constant
parameters: k1 = 1 nM/minute
K2 = 0 1/minute, []constant
k3 = 0.005 1/(nM*minute)
```

K2 is a variable rate parameter whose value is defined by an algebraic rule. See "Algebraic Rule 17, Rate Parameter K2" on page C-43. Its value varies from 0.005 to 0.251/minute.

#### Rate Rule 2, M-Phase Promoting Factor (MPF)

The rate rule is from "Equation 2, M-Phase Promoting Factor" on page C-30.

K2, Kcdc25, and Kwee1 are variable rate parameters whose values are defined by algebraic rules. See "Algebraic Rule 17, Rate Parameter K2" on page C-43, "Algebraic Rule 18, Rate Parameter Kcdc25" on page C-43, and "Algebraic Rule 19, Rate Parameter Kwee1" on page C-43.

#### Rate Rule 3, Inhibited M-Phase Promoting Factor (pMPF)

The rate rule is from "Equation 3, Inhibited M-Phase Promoting Factor" on page C-31.

```
rate rule: pMPF = Kwee1*MPF - (Kcdc25 + kcak + K2)*pMPF + kpp*pMPFp
```

# Rate Rule 4, Activated but Inhibited M-Phase Promoting Factor (pMPFp)

The rate rule is from "Equation 4, Inhibited and Activated M-Phase Promoting Factor" on page C-31.

```
rate rule: pMPFp = Kwee1*MPFp - (kpp + Kcdc25 + K2)*pMPFp + kcak*pMPF
```

#### Rate Rule 5, Activated M-Phase Promoting Factor (MPFp)

The rate rule is from "Equation 5, Activated M-Phase Promoting Factor" on page C-32.

```
rate rule: MPFp = kcak*MPF - (kpp + Kwee1 + K2)*MPFp + Kcdc25*pMPFp
```

#### Rate Rule 11, Activated Cdc25 (Cdc25p)

The rate rule is from "Equation 11, Cell Division Control 25" on page C-33.

#### Rate Rule 12, Inhibited Wee1 (Wee1p)

The rate rule is from "Equation 12, Wee1 Activation/Deactivation" on page C-33.

## Rate Rule 13, Activated Intermediate Enzyme (IEp)

The rate rule is from "Equation 13, Intermediate Enzyme Activation/Deactivation" on page C-34.

## Rate Rule 14, Activated APC (APCa)

The rate rule is from "Equation 14, APC Activation/Deactivation" on page C-34.

### Algebraic Rule 17, Rate Parameter K2

K2 is a variable rate parameter whose value is determined by the amount of active and inactive APC. The algebraic rule is from "Equation 17, Rate Parameter K2" on page C-35.

```
algebraic rule: V2i*(TotalAPC - APCa) + V2a*APCa - K2
species: APCi = 1 nM
APCa = 0 nM
TotalAPC = 1 nM [x]constant
parameters: K2 = 0 or 0.25 1/minute, []constant
V2i = 0.005 1/(nM*minute)
V2a = 0.25 1/(nM*minute)
```

#### Algebraic Rule 18, Rate Parameter Kcdc25

Kcdc25 is a variable rate parameter whose value is determined by the amount of active and inactive Cdc25. The algebraic rule is from "Algebraic Rule 18, Rate Parameter Kcdc25" on page C-43.

algebraic rule: V25i\*(TotalCdc25 - Cdc25p) + V25a\*Cdc25p - Kcdc25

#### Algebraic Rule 19, Rate Parameter Kwee1

Kwee1 is a variable rate parameter whose value is determined by the amount of active and inactive Wee1. The algebraic rule is from "Equation 19, Rate Parameter Kwee1" on page C-36.

algebraic rule: Vweei\*Wee1p + Vweea\*(TotalWee1 - Wee1p) - Kwee1

# SimBiology Model with Reactions and Algebraic Rules

- "Overview" on page C-45
- "Reaction 1, Synthesis of Cyclin B" on page C-45
- "Reaction 2, Degradation of Cyclin B" on page C-46

С

- "Reaction 3, Dimerization of Cyclin B with Cdc2 Kinase" on page C-47
- "Reaction 4, Degradation of Cyclin B on MPF" on page C-47
- "Reaction 5, Deactivation of Active MPF" on page C-48
- "Reaction 6, Activation of MPF" on page C-49
- "Reaction 7, Remove Inhibiting Phosphate from Inhibited MPF" on page C-50
- "Reaction 8, Inhibition of MPF by Phosphorylation" on page C-51
- "Reaction 11, Degradation of Cyclin B on Inhibited MPF" on page C-53
- "Reaction 12, Deactivation of MPF to Inhibited MPF" on page C-53
- "Reaction 13, Activation of Inhibited MPF" on page C-53
- "Reaction 15, Degradation of Cyclin B on Active but Inhibited MPF" on page C-54
- "Reaction 16, Inhibit MPF by Phosphorylation" on page C-54
- "Reaction 17, Remove Inhibiting Phosphate from Activated MPF" on page C-55
- "Reaction 19, Degradation of Cyclin B on Activated MPF" on page C-55
- "Reaction 36, Activation of Cdc25 by Activated MPF" on page C-55
- "Reaction 37, Deactivation of Cdc25" on page C-56
- "Reaction 38, Deactivation of Wee1 by Active MPF" on page C-56
- "Reaction 39, Activation of Wee1" on page C-56
- "Reaction 40, Activation of Intermediate Enzyme by Active MPF" on page C-57
- "Reaction 41, Deactivation of IE" on page C-57
- "Reaction 42, APC Activation by IEp" on page C-57
- "Reaction 43, APC Deactivation" on page C-57
- "Block Diagram of the M-Phase Control Model with Reactions" on page C-58

### Overview

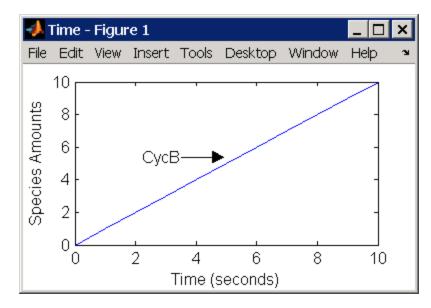
There can be one or more reactions for an equation defining a species and one algebraic rule for each variable parameter in the M-phase control model [Marlovits 1998]. For a list and description of the equations, see "M-Phase Control Equations" on page C-27.

A basic model includes reactions 1 to 8, 11 to 13, 15 to 17, 19, and 36 to 43 with algebraic rules from equations 17, 18, and 19.

## **Reaction 1, Synthesis of Cyclin B**

Cyclin B is synthesized at a constant rate.

Simulate reaction 1 with the sundials solver.



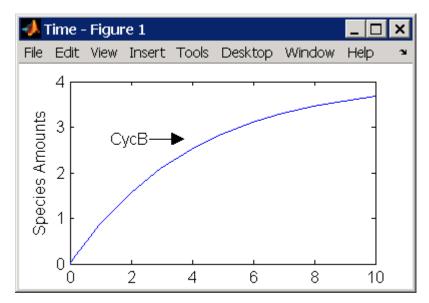
## Reaction 2, Degradation of Cyclin B

Cyclin B is degraded at the end of the M-phase.

```
reaction: CycB -> AA
reaction rate: K2*CycB nM/minute
parameters: K2 = 0 1/minute, []constant, variable by rule
V2i = 0.005 1/nM*minute
V2a = 0.25 1/nM*minute
species: CycB = 0 nM
APCi = 1 nM
APCa = 0 nM
AA = 100 nM [x]constant [x]boundary
algebraic rule: (V2i*APCi) + (V2a*APCa) - K2
```

Initially, Cyclin B degradation is low. This implies the amount of active APC (APCa) = 0 and inactive APC (APCi) = APCtotal = 1 nM.

Test the algebraic rule by simulating reactions 1 and 2 with APCi = 0 and APCa = 1.

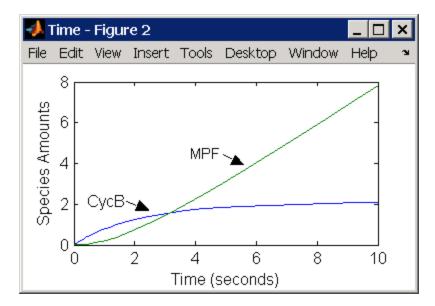


## Reaction 3, Dimerization of Cyclin B with Cdc2 Kinase

Cyclin B dimerizes with Cdc2 kinase to form M-phase promoting factor (MPF).

```
reaction: Cdc2 + CycB -> MPF
reaction rate: k3*Cdc2*CycB nM/minute
parameters: k3 = 0.005 1/(nM*minute)
species: Cdc2 = 100 nM
CycB = 0 nM
MPF = 0 nM
```

Test the model by simulating with K2 = 0.25.

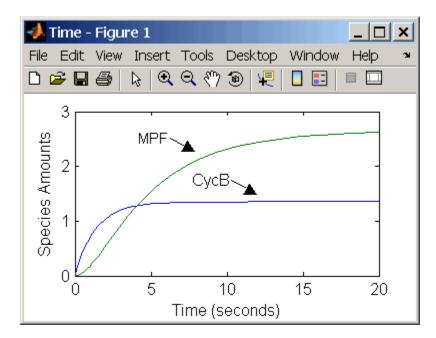


#### **Reaction 4, Degradation of Cyclin B on MPF**

Cyclin B is tagged with ubiquitin groups and degrades while bound to Cdc2.

```
APCi = 1 nM
APCa = 0 nM
AA = 100 nM [x]constant [x]boundary
algebraic rule: (v2i*APCi) + (v2a*APCa) - K2
```

Test the simulation with APCa = 1 and APCi = 0. Because the amount of APCa (active) is high, K2 increases and the degradation starts to balance the synthesis of MPF.



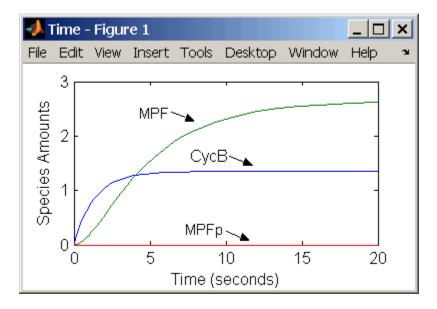
#### **Reaction 5, Deactivation of Active MPF**

Active MPF (MPFp) is dephosphorylated on Thr-161 by an unknown phosphatase (PP) to inactive MPF (MPF).

```
reaction: MPFp -> MPF
reaction rate: kpp*[MPFp]
parameters: kpp = 0.004 1/minute
species: MPFp = 0 nM
MPF = 0 nM
```

kcakr = 0.004 1/minute [Marlovits 1998, p. 175], but is renamed to kpp [Borisuk 1998].

Test simulation with APCa = 1 and APCi = 0. MPF increases without reaching steady state.



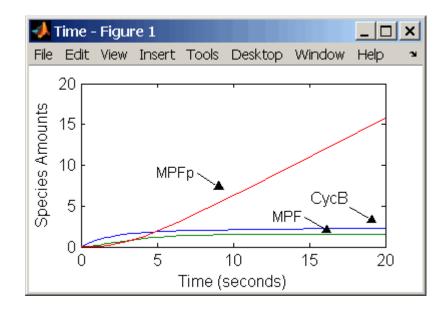
#### **Reaction 6, Activation of MPF**

Inactive MPF (MPF) is phosphorylated on Thr-161 by an unknown cyclin activating kinase (CAK).

```
reaction: MPF -> MPFp
reaction rate: kcak*[MPF]
parameters: kcak = 0.64 1/minute
species: MPF = 0 nM
MPFp = 0 nM
```

The kinase reaction that phosphorylates MPF to the active form is 160 times faster than the phosphatase reaction that dephosphorylates active MPF.

Simulate the model with reactions 1 to 6. Notice that after adding reaction 6, most of the product goes to active MPF (MPFp).



#### Reaction 7, Remove Inhibiting Phosphate from Inhibited MPF

Cdc25 phosphatase removes the inhibiting phosphate groups at the threonine 14 and tyrosine 15 residues on Cdc2 kinase.

Initially, all of the Cdc25 phosphatase is in the inactive form (Cdc25).

Enter the initial value for Kcdc25 as 0.0 and let the first time step calculate the value from the rule, or enter an initial value using the rule.

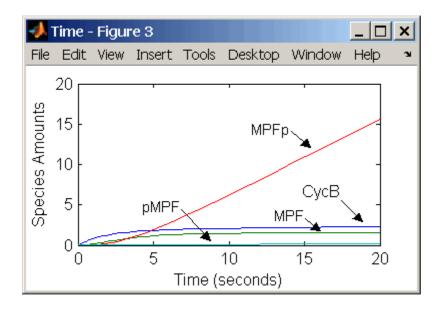
Initially, set **ConstantAmount** for Cdc25 and Cdc25p to test reactions 1 through 7. Then after you can add the reactions to regulate the Cdc25 phosphatase by clearing the **ConstantAmount** property.

## Reaction 8, Inhibition of MPF by Phosphorylation

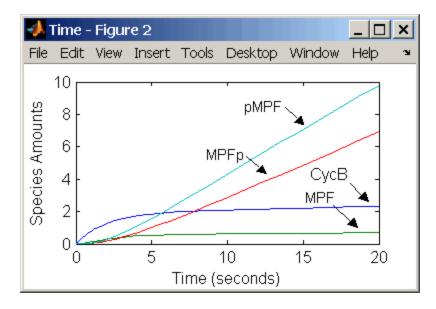
Addition of inhibiting phosphate groups by Wee1 kinase to inhibit active M-phase promoting factor (MPF). Myt1 kinase is also involved with the phosphorylation, but its contribution is grouped with Wee1.

The initial capitalization for the parameter Kwee1 is a convention to indicate that this value changes during the simulation.

Test the simulation for reactions 1 through 8 with Wee1p (inactive) = 1 and Wee1 (active) = 0.



Test the simulation with Wee1p (inactive) = 0 and Wee1 (active) = 1.



C-52

#### Reaction 11, Degradation of Cyclin B on Inhibited MPF

Degradation of cyclin B (CycB) on inhibited MPF (pMPF). Cyclin B is tagged with ubiquitin groups and degrades while bound to Cdc2.

```
reaction: pMPF -> Cdc2 + AA
reaction rate: K2*[pMPF] nM/minute
parameters: K2 = 0 or 0.25 1/minute, variable by rule
V2i = 0.005 1/nM*minute
V2a = 0.25 1/nM*minute
species: MPF = 0 nM
APCi = 1 nM
APCa = 0 nM
AA = 100 nM [x]constant [x]boundary
Cdc2 = 100 nm
algebraic rule: (V2i*APCi) + (V2a*APCa) - K2
```

Test the simulation with Weel active (Wee1 = 1) and APC active (APCi = 1).

#### Reaction 12, Deactivation of MPF to Inhibited MPF

Inhibited/active MPF (pMPFp) is dephosphorylated on Thr-161 by an unknown phosphatase (PP) to inhibited MPF (pMPF). Compare reaction 12 with reaction 5.

```
reaction: pMPFp -> pMPF
reaction rate: kpp*[pMPFp]
parameters: kpp = 0.004 1/minute
species: pMPFp = 0 nM
pMPF = 0 nM
```

#### **Reaction 13, Activation of Inhibited MPF**

Inhibited MPF (pMPF) is phosphorylated on Thr-161 by an unknown cyclin-activating kinase (CAK). Compare reaction 13 with reaction 6.

```
reaction: pMPF -> pMPFp
reaction rate: kcak*[pMPF] nM/minute
parameters: kcak = 0.64 1/minute
species: pMPF = 0 nM
pMPFp = 0 nM
```

Test the simulation with Wee1p = 1 (inactive)/ Wee1 = 0 and then test with Wee1p = 0 (inactive)/ Wee1 = 1.

# Reaction 15, Degradation of Cyclin B on Active but Inhibited MPF

Degradation of cyclin B (CycB) on inhibited MPF (pMPF). Cyclin B is tagged with ubiquitin groups and degrades while bound to cdc2 kinase.

```
reaction: pMPFp -> Cdc2 + AA
reaction rate: K2*[pMPFp] nM/minute
parameters: K2 = 0 or 0.25 1/minute, variable by rule
v2i = 0.005 1/nM*minute
v2a = 0.25 1/nM*minute
species: MPF = 0 nM
APCi = 1 nM
APCa = 0 nM
AA = 100 nM [x]constant [x]boundary
Cdc2 = 100 nm
algebraic rule: (V2i*APCi) + (V2a*APCa) - K2
```

#### Reaction 16, Inhibit MPF by Phosphorylation

Addition of inhibiting phosphate groups by Wee1 kinase to inhibit active M-phase promoting factor (MPF). Myt1 kinase is also involved with the phosphorylation, but its contribution is grouped with Wee1.

```
reaction: MPFp -> pMPFp
reaction rate: Kwee1*[MPFp] nM/minute
parameters: Kwee1 = 1/minute []constant, variable by rule
Vweei = 0.01 1/nM*minute
Vweea = 1 1/nM*minute
species: MPFp = 0 nM
pMPFp = 0 nM
Wee1p = 1 nM (inactive)
Wee1 = 0 nM (active)
algebraic rule: (Vwee1i*Wee1p) + (Vwee1a*Wee1) - Kwee1
```

#### Reaction 17, Remove Inhibiting Phosphate from Activated MPF

Remove the inhibiting phosphate group from pMPFp with cdc25 phosphatase.

```
reaction: pMPFp -> MPFp
reaction rate: Kcdc25*[pMPFp]
parameters: Kcdc25 = 0 1/minue, []constant, variable by rule
V25i = 0.017 1/nM*minute
V25a = 0.17 1/nM*minute
species: pMPFp = 0 nM
MPFp = 0 nM
algebraic rule: (V25i*Cdc25) + (V25a*Cdc25p) - Kcdc25
```

### Reaction 19, Degradation of Cyclin B on Activated MPF

Degradation of cyclin B (CycB) on inhibited MPF (pMPF). Cyclin B is tagged with ubiquitin groups and degrades while bound to cdc2 kinase.

```
reaction: MPFp -> MPF + AA
reaction rate: K2*[MPFp] nM/minute
parameters: K2 = 0 or 0.25 1/minute, variable by rule
V2i = 0.005 1/nM*minute
V2a = 0.25 1/nM*minute
species: MPF = 0 nM
MPFp = 0 nM
APCi = 1 nM
APCa = 0 nM
AA = 100 nM [x]constant [x]boundary
Cdc2 = 100 nm
algebraic rule: (V2i*APCi) + (V2a*APCa) - K2
```

#### Reaction 36, Activation of Cdc25 by Activated MPF

Activation of cdc25 phosphatase by phosphorylation with active M-phase promoting factor (MPFp).

species: Cdc25 = 1 nM (inactive)
 Cdc25p = 0 nM (active)

Initially MPF is inhibited (MPF\* reacts to pMPF\*).

#### Reaction 37, Deactivation of Cdc25

Deactivation of cdc25 phosphatase by dephosphorylation with an unknown phosphatase.

```
reaction: Cdc25p -> Cdc25
reaction rate: (k25r*[Cdc25p])/(Km25r + [Cdc25p])
parameters: k25r = 0.1 nM/minute
        Km25r = 1 nM
        species: Cdc25 = 1 nM (inactive)
        Cdc25p = 0 nM (active)
```

#### Reaction 38, Deactivation of Wee1 by Active MPF

Deactivation of Wee1 kinase by phosphorylation with active M-phase promoting factor (MPFp).

Initially MPF is inhibited (MPF\* reacts to pMPF\*).

#### Reaction 39, Activation of Wee1

Activation of Wee1 kinase by dephosphorylation with an unknown kinase.

Wee1 = 0 nM (active)

#### Reaction 40, Activation of Intermediate Enzyme by Active MPF

The inactive intermediate enzyme (IE) is activated by phosphorylation with active M-phase promoting factor (MPFp).

```
reaction: IE + (MPFp) -> IEp + (MPFp)
reaction rate: (kie*[MPFp]*[IE])/(Kmie + [IE])
parameters: kie = 0.02 1/minute
    Kmie = 0.01nM
    species: IE = 1 nM (inactive)
    IEp = 0 nM (active)
```

#### Reaction 41, Deactivation of IE

The active intermediate enzyme (IE) is deactivated by dephosphorylation.

```
reaction: IEp -> IE
reaction rate: (kier*[IEp])/(Kmier + [IEp])
parameters: kier = 0.15 nM/minute
        Kmier = 0.01 nM
species: IE = 1 nM (inactive)
        IEp = 0 nM (active)
```

#### **Reaction 42, APC Activation by IEp**

Anaphase-promoting complex (APC) is activated by an active intermediate enzyme (IEp).

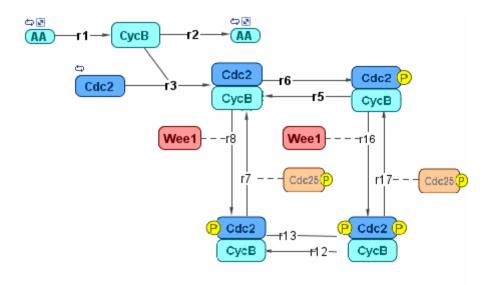
```
reaction: APCi + IEp -> APCa + IEp
reaction rate: (kap*[IEp]*[APCi])/(Kmap + [APCi])
parameters: kap = 0.13 1/minute
        Kmap = 0.01 nM
    species : APCi = 1 nM
        APCa = 0 nM
```

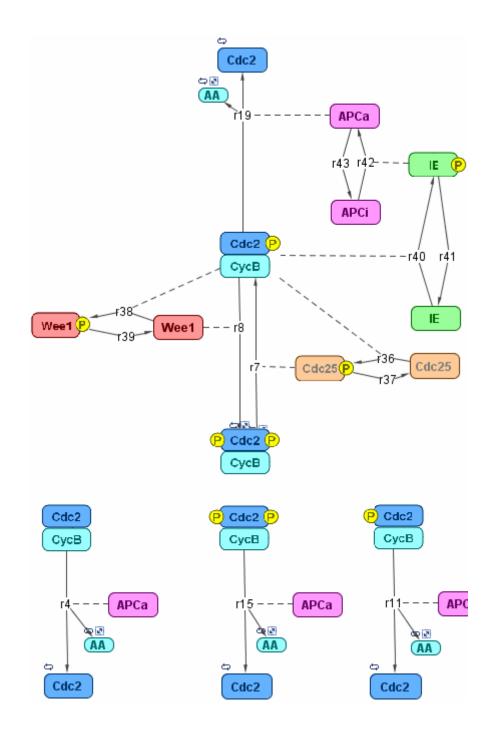
#### **Reaction 43, APC Deactivation**

Anaphase-promoting complex (APC) is deactivated.

```
reaction: APCa -> APCi
reaction rate: (kapr*[APCa])/(Kmapr + [APCa])
parameters: kapr = 0.13 nM/minute
        Kmapr = 1 nM
    species : APCi = 1 nM
        APCa = 0 nM
```

#### **Block Diagram of the M-Phase Control Model with Reactions**





## References

[1] Borisuk M, Tyson J (1998), "Bifurcation analysis of a model of mitotic control in frog eggs," Journal of Theoretical Biology, 195(1):69–85, PubMed 9802951.

[2] Marlovits G, Tyson C, Novak B, Tyson J (1998), "Modeling M-phase control in Xenopus oocyte extracts: the surveillance mechanism for unreplicated DNA," Biophysical Chemistry, 72(1-2):169–184, PubMed 9652093.

[3] Novák B, Tyson J (1993), "Numerical analysis of a comprehensive model of M-phase control in Xenopus oocyte extracts and intact embryos," Journal of Cell Science, 106(4):1153–1168, PubMed 8126097.

# Index

# A

algorithm explicit tau-leaping 3-6 3-8 implicit tau-leaping 3-9 SSA 3-8 analysis conserved moieties 3-1 parameter estimation 3-1 parameter fitting 4-1 pharmacokinetics 4-1 sensitivity 3-1

## C

conserved moieties 2-6 covariate analysis 4-41

## E

explicit tau-leaping algorithm 3-8

## G

G protein cycle background C-19 in yeast C-19 model C-19 modeling of C-20 G protein cycle model experimental data and assumptions C-20 reactions overview C-20 G proteins introduction C-19

## 

implicit tau-leaping algorithm 3-9

#### Μ

model

pharmacokinetic 4-1 models G protein cycle C-19 moiety conservation 2-6

#### Ν

NLME 4-2

#### Ρ

parameter estimation 3-29 parameter scan 3-27 parameters estimation of 3-1 4-1

## R

references yeast G protein cycle model C-24

## S

scan parameters 3-27 sensitivity analysis 3-18 SimBiology simulation overview 3-3 stochastic (SSA) algorithm 3-8 stochastic solvers explicit tau-leaping algorithm 3-6 implicit tau-leaping algorithm 3-6 references 3-6 SSA 3-6

#### Y

yeast G protein cycle model experimental data and assumptions C-20 graphical representation C-20 introduction C-19 to C-20 reactions overview C-20 references C-24